

EXCHANGE RATE **VOLATILITY**, FINANCIAL **CRISIS** AND LARGE **OUTPUT** LOSS: STYLIZED **FACTS** AND **SOME** USEFUL **THEORIES**

WILLI SEMMLER*

In recent times certain countries or regions have experienced severe financial crises with large output loss. This has occurred whether the exchange rates were pegged or flexible. There appear to be basic mechanisms at work from which even flexible exchange rate regimes cannot escape. This paper reviews some of the stylized facts that appear to be common to such financial crises and surveys some recent financial market models that attempt to model such macro-caused financial and real crises. We focus on the connection between exchange rate volatility, financial crisis and large output loss. We find models that exhibit multiple equilibria particularly relevant to explain macro-caused financial and real crises.

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Introduction

With the end of the Bretton Woods system in the 1970s and the financial market liberalization in the 1980s and 1990s the international economy has experienced several financial crises in certain countries or regions entailing, in most cases, large output losses. This has occurred whether the exchange rates were pegged or flexible. There appear to be basic mechanisms at work from which even flexible exchange rate regimes cannot escape. This paper will review some of the stylized facts that appear to be common to such financial crises and surveys some recent financial market models that attempt to model such macro-caused financial and real crises. We focus on the connection between exchange rate volatility, financial crisis and large output loss.

With respect to exchange rates and financial crisis three views, in fact three generations of models, have been presented in the literature. A first view maintains that news on macro-economic fundamentals (differences in economic growth rates, productivity differences and differences in price levels, in the short run interest rates as well as in monetary policy actions) cause exchange rate movements. The second view maintains that speculative forces drive exchange rates where there can be self-fulfilling expectations at work destabilizing exchange rates without deterioration of fundamentals. Third, following the theory of imperfect capital markets, it has also been maintained that the dynamics of self-fulfilling expectations depend on some fundamentals for example, the strength and weakness of the balance sheets of the economic units such as households, firms, banks and governments. From the latter point of view we can properly study the connection between deterioration of fundamentals, exchange rate volatility and financial instability of a country. Although, recently there have been proposed diverse micro as well macroeconomic theories to explain financially caused recessions we think that those types of models are particularly relevant that exhibit multiple equilibria, because they appear to be particularly suited to model recent financially caused large output losses.

** University of Bielefeld and New School University, 65 Fifth Avenue, New York, NY 10003, USA. This paper is based on the book manuscript by Semmler and Wöhrmann (1999) where a more extensive presentation of the topics discussed here can be found. More technical versions of this paper have been presented at workshops at the University of Technology, Vienna, University of Antwerpen, University of Leuven and the macroworkshop, Columbia University, New York.*

Stylized Facts

There have been three major episodes of international financial crisis for certain regions or countries entailing a large output loss. These were 1) the 1980s Latin American debt crisis, 2) the 1994-95 Tequila Crisis (Mexico, Argentina), 3) the 1997-98 Asian financial crisis (as well as the Russian financial crisis 1998). To study such crises we will look at the interplay of exchange rates, financial markets, severe reversal of financial flows and large output losses.

Central in this context will be the balance sheets of firms, households, banks and governments. Weak balance sheets of those economic units mean that liabilities are not covered by assets. In particular heavy external liabilities of economic units such as firms, banks or countries can cause a sudden reversal of capital flows initiating a currency crisis. Exchange rate risk and a sudden reversal of capital flows is often built up by a preceding increase of insolvency risk of a country. The deterioration of balance sheets of households, firms and banks often have come about by a preceding lending boom and increased risk taking. A currency crisis is likely to entail a rise in the interest rate, a stock market crash and a banking crisis. Yet, financial and exchange rate volatility is not always leading to interest rate increase and stock market crash. It is thus not necessary that a financial instability will be propagated. The major issue in fact is what the assets of the economic units represent. If economic units borrow against future income streams they may use net worth as collateral. The wealth of the economic units, or of a country of that matter, are the discounted future income streams. Sufficient net wealth makes the agents solvent otherwise they are threatened by insolvency which is equivalent to saying that the liabilities outweigh the assets. The question is only what are good proxies to measure insolvency, i.e. what is sustainable debt.¹ But, of course, exchange rate volatility and curren-

¹ In Semmler and Sieveking [1999] a procedure is proposed of how to determine and to estimate sustainable debt. A sketch of the econometric estimation procedure is given in the appendix of this paper. For debt dynamics in a macro model, see Flaschel *et al.* [2000, ch. 3].

cy crisis are relevant factors as well and thus what determines exchange rate movements.

There are typical stylized facts to be observed before and after the financial crises which have been studied by numerous papers [see for example Mishkin 1998, Milesi-Ferreti and Razen, 1996, 1998]. Empirical literature on financial crisis episodes may allow us to summarize the following stylized facts:

- there is frequently a deterioration of balance sheets of economic units (households, firms, banks, the government and the country)
- before the crises the current account deficit to GDP ratio rises
- preceding the currency crisis the external debt to reserves ratio rises (after the crisis the current account recovers)
- there is a sudden reversal of capital flows and unexpected depreciation of the currency
- domestic interest rates jump up (partly initiated by central banks policy)
- subsequently the stock prices fall
- a banking crisis occurs with large loan losses by banks and subsequent contraction of credit (sometimes moderated by a bail out of failing banks by the government)
- the financial crisis follows a large output loss due to large scale bankruptcies of firms and financial institutions

Since most recent financial crises were indeed triggered by a sudden reversal of capital flows and an unexpected depreciation of the currency (partly caused by deteriorating fundamentals, such as balance sheets of agents and current account deficit, rising foreign debt and declining short term debt to reserve ratio) we will first consider the traditional exchange rate model to study how it helps us to understand the above financial crisis mechanism.

The Standard Exchange Rate Overshooting Model

In earlier work starting with Dornbusch's seminal paper on open economy dynamics [1976] and in following contributions by other authors, the economy is stylized in a

very simple way through an asset market and a product market. The asset market, represented by the money market, is always at a temporary equilibrium which clears by the fast adjustment of the nominal interest rate. In the product market, prices are postulated to adjust in a Walrasian fashion. In flex-price models the temporary equilibrium in the product market is established through the fast adjustment of prices. On the other hand, it is often assumed that prices are sticky or prices move only sluggishly. In this section we consider the case when output is fixed and prices move. In section 4 we study a model of the IS-type where prices are sticky and output moves.

Dornbusch's original version belongs to the first variant. His model as well as subsequent papers employ a differential equation approach to formulate the exchange rate and the price dynamics. With the assumption of perfect foresight, the change of the expected exchange rate is then equated with the right hand derivative of the actual exchange rate. This assumption is related to the interest rate parity theory. The same is proposed, where taken up, for the expected price change.

A number of variations of this general approach can be found in the literature. For details of such models and their critical evaluation, see Flaschel, Franke and Semmler [1997].

The dynamics of perfect foresight rational expectations models are characterized by saddle path stability. Small displacements from the equilibrium path will give rise to unstable dynamics. In these models it is then postulated that the variable in question –the exchange rate or price level– will always jump back to the stable path, in more technical terms, to the stable manifold which secures that the transversality condition holds. What the observer would thus see is some jump or overshooting of exchange rates when there are some news to fundamentals observed. Due to this exchange rate (or other asset prices, if they are in the model) may fluctuate or even be volatile.

Let us study the basic exchange rate overshooting model more formally. Dornbusch [1976] and Gray and Turnovsky [1979] have provided us with basic models of exchange

rate volatility. Here only simple domestic foreign assets are considered. Moreover, borrowing and lending and credit market are left aside as well. There is only domestic and foreign currency.

As above mentioned the traditional exchange rate model results in saddle path stability under perfect foresight using interest parity theory. To explain this model we use the following notations:

i = domestic interest rate; i^* = foreign interest rate; x = expected rate of exchange rate depreciation; e = current exchange rate; M = log of domestic money supply; p = log of price level; Y = log of output

$$i = i^* + x \quad (1a)$$

$$x = \dot{e} \quad (\text{perfect foresight}) \quad (1b)$$

$$M - p = \alpha_1 Y + \alpha_2 i \quad \alpha_1 > 0, \alpha_2 < 0 \quad (2)$$

$$\dot{p} = p [\beta_0 + (\beta_1 - 1) Y + \beta_2 i + \beta_3 (e - p)] \quad (3)$$

$$0 < \beta_1 < 1; \beta_2 < 0; \beta_3 > 0, p > 0$$

The equilibrium is:

$$\bar{i} = i^*$$

$$\bar{x} = 0$$

$$\bar{M} - \bar{p} = \alpha_1 Y + \alpha_2 \bar{i}$$

$$\beta_0 + (\beta_1 - 1) Y + \beta_2 \bar{i} + \beta_3 (\bar{e} - \bar{p}) = 0$$

thus:

$$d\bar{p} = d\bar{e} = d\bar{M}$$

We obtain the following dynamics:

from (1a) and (1b) we get

$$\dot{e} = i(M, p, Y) - i^* \quad (4)$$

and from (2) we obtain

$$i(M, p, Y) = \frac{M - p - \alpha_1 Y}{\alpha_2} \quad (5)$$

Therefore, we have, as differential equations, (4) and the following (6)

$$\dot{p} = p [\beta_0 + (\beta_1 - 1) Y + \beta_2 i + \beta_3 (e - p)] \quad (6)$$

Eqs. (4) and (6) are our two differential equations which exhibit saddle point instability (for details, see Gray and Turnovsky, 1979).

$$\begin{pmatrix} \dot{e} \\ \dot{p} \end{pmatrix} = \begin{pmatrix} 0 - 1/\alpha_2 \\ \rho\beta_3 - \rho(\beta_3 + \beta_2/\alpha_2) \end{pmatrix} \begin{pmatrix} e \\ p \end{pmatrix} + \begin{pmatrix} 1/\alpha_2 \\ \rho\beta_2/\alpha_2 \end{pmatrix} m(t) \quad (7)$$

drift term (can be neglected for the local dynamics)

Here, however, e is not free to jump instantaneously to the stable branch of the saddle paths. Thus, the usual jump variable technique is applied:

“This frees e to jump at time zero, thereby rendering the predetermined value e_0 irrelevant for the future evolution of the system” [Gray and Turnovsky, 1979:649] “...we find that an important role in the solution procedure is played by the transversality conditions... The effect of imposing these conditions is typically to force the system on to the stable arm of the saddle, thereby ensuring stability of the resulting dynamic system” [p. 650].

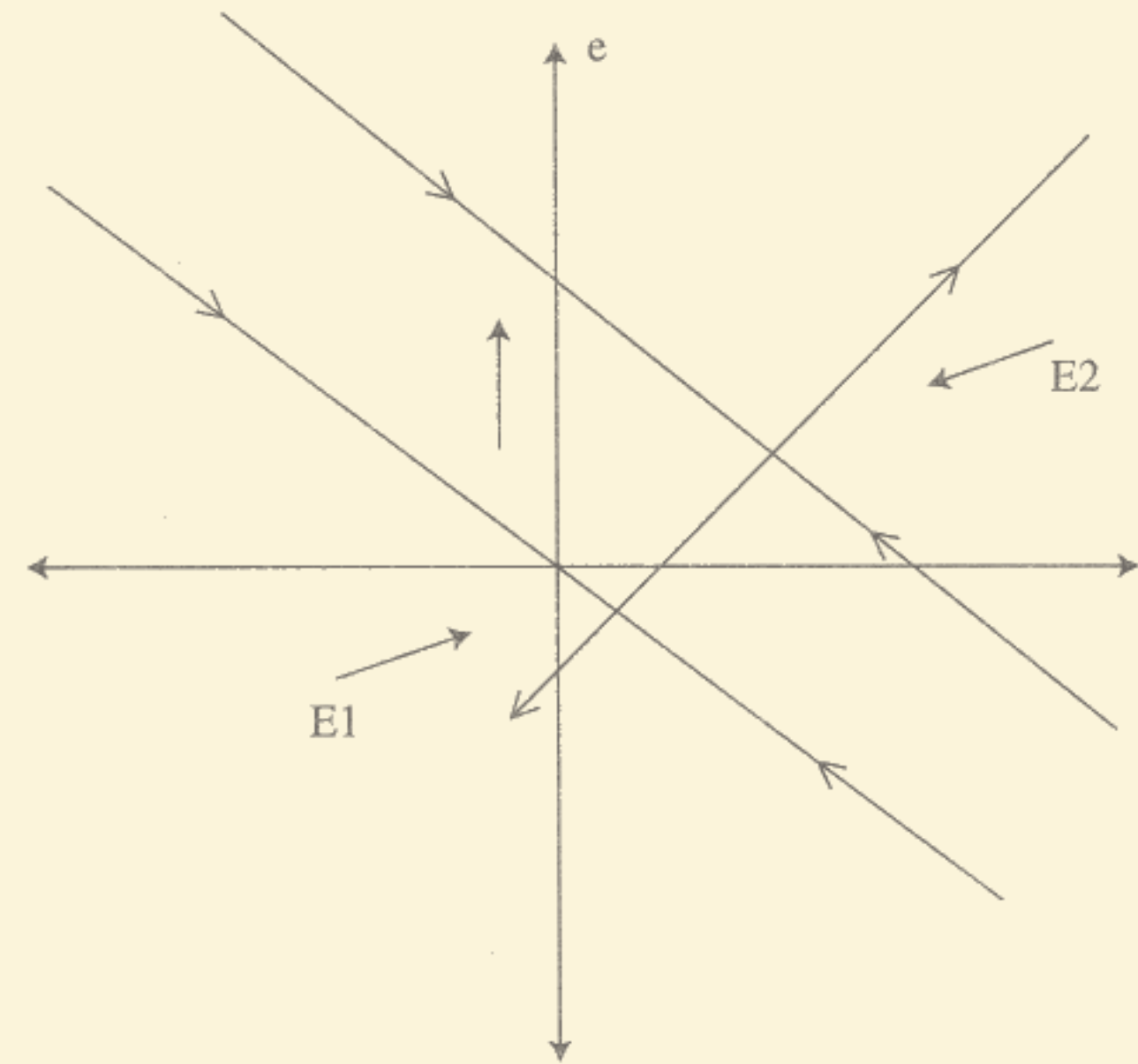
We want to note that first an increase in money supply makes e jumping up and then slowly moving down to E_2 (with prices then increasing).

Note also that the product market is in disequilibrium, but price movement equilibrates it. Yet, we could also assume that output changes, if prices are sticky a model to be considered in the next section.

Let us now consider a financial crisis in the context of an open economy with a flexible exchange rate system. We

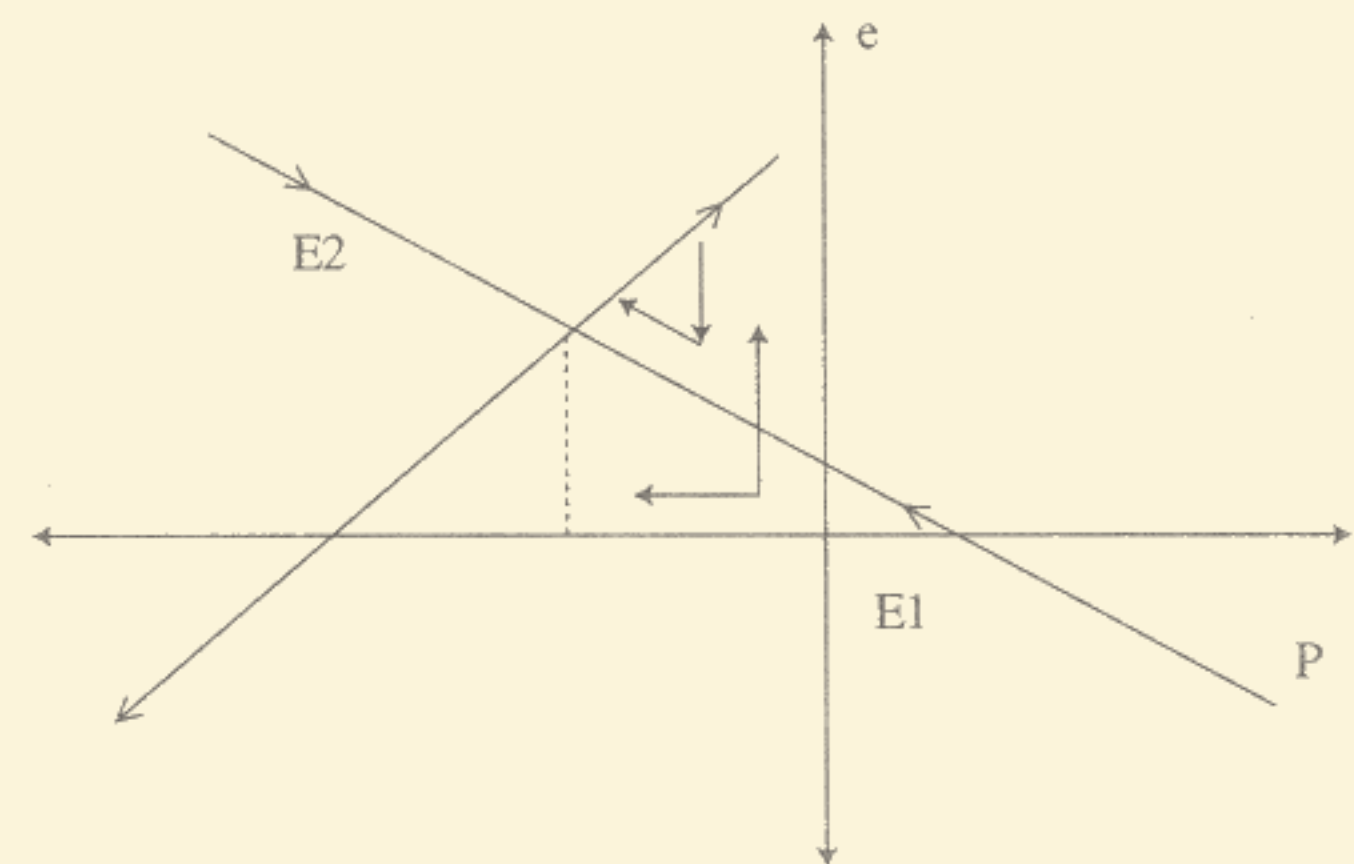
start with the following modification of the overshooting model, again leaving aside other asset and the credit markets.

Figure 1
Illustration of the jump variable technique



The financial crisis in the framework of the overshooting model could then look like:

Figure 2
Financial crisis in an overshooting model



We have posited the following sequence:

1. sudden depreciation of the currency due to an increase of risk (= R), to be included in equ. (1a, b).
2. central bank decreases the money supply (increase the interest rate).
3. exchange rate has been overshooting but jumps back to the stable branch and moves to E_2 .

Therefore, given equ.

$$3) \dot{p} = \rho [\beta_0 + (\beta_1 - 1) Y + \beta_2 i + \beta_3 (e - p)], \quad (8)$$

demand will contract (because i increases) and prices will fall. On the other hand the increase in e has only a small effect on the increase in demand (a depreciation will only slowly increase demand).

Such treatment of exchange rates –through of perfect foresight rational expectations models– have been called into question [see Flaschel, Franke and Semmler, 1997]. A variable's jump to the stable manifold requires a lot of information for the agents. Stiglitz has always argued that there are no conceivable market adjustment processes that could allow for such a fast adjustment to the stable branch. In addition, there is an absence of convincing empirical evidence in support of such jumps. In light of these shortcomings, recently economists prefer employ adaptive learning procedures to explain the convergence to the stable branch. Such mechanism are then supposed to explain whether and how a rational expectations path is reached. The development of the econometrics of ARMA processes has strongly strengthened this direction of research. Small-scale macrodynamic models in which the right hand derivative of the price level is replaced by one or multi period ahead forecast of endogenous variables (or learning mechanism) have already been studied. In Adelzadeh and Semmler [1996] a model is constructed and an econometric learning procedure is utilized for the forecast of the exchange rate which avoids the difficulties of the perfect foresight versions of rational expectations models. The procedure does not require the variable under study to be always on the stable manifold (or to get back to it through jumps). The recursive procedure iteratively allows for the adaptive learning of forecasted endogenous variables. One can fruitfully use those learning procedures to understand exchange rate dynamics in open economies.

Yet, note that the overshooting model has in place of an is equation an equation for a price dynamics, see equ. (3). In equ. (3) output is fixed. This and the missing asset markets

might not be very realistic features of the model and will be relaxed in the next model.

Exchange Rates, Balance Sheets and Multiple Equilibria

The work by Krugman has been particularly useful in modeling exchange rate volatility, financial instability and financially caused recessions in IS-LM type of models. Krugman has been involved in elaborating on the three generations of models that were mentioned in the introduction.

Recently Krugman [1999a, 1999b] presented some further work and developed extensions of the IS-LM model that include exchange rates, debt dynamics and output dynamics. He has particularly stressed the importance of the balance sheets of economic agents (banks, households, and firms) for macrodynamics. As in Mishkin [1998], with sound balance sheets of banks, firms and households exchange rate or financial shocks do not translate into a deep financially caused recessions. Weak balance sheets are vulnerable to shocks and can be translated into large output losses. This result is obtained in a model of multiple equilibria. Central in the Krugman models is the debt denominated in foreign currency as a fraction to total debt. Firms need collateral for borrowing. With low collaterals they are likely to receive less credit. When an exchange rate shock occurs the debt denominated in foreign currency rises, the debt service obligation of firms, households and banks rise and –due to the loss of collaterals– they receive less credit. Formally the Krugman [1999a, 1999b] models suggest a modification of the traditional IS-model. The traditional IS-model reads

$$Y = D(Y, i) + NX(eP^*/P, Y) \quad (9)$$

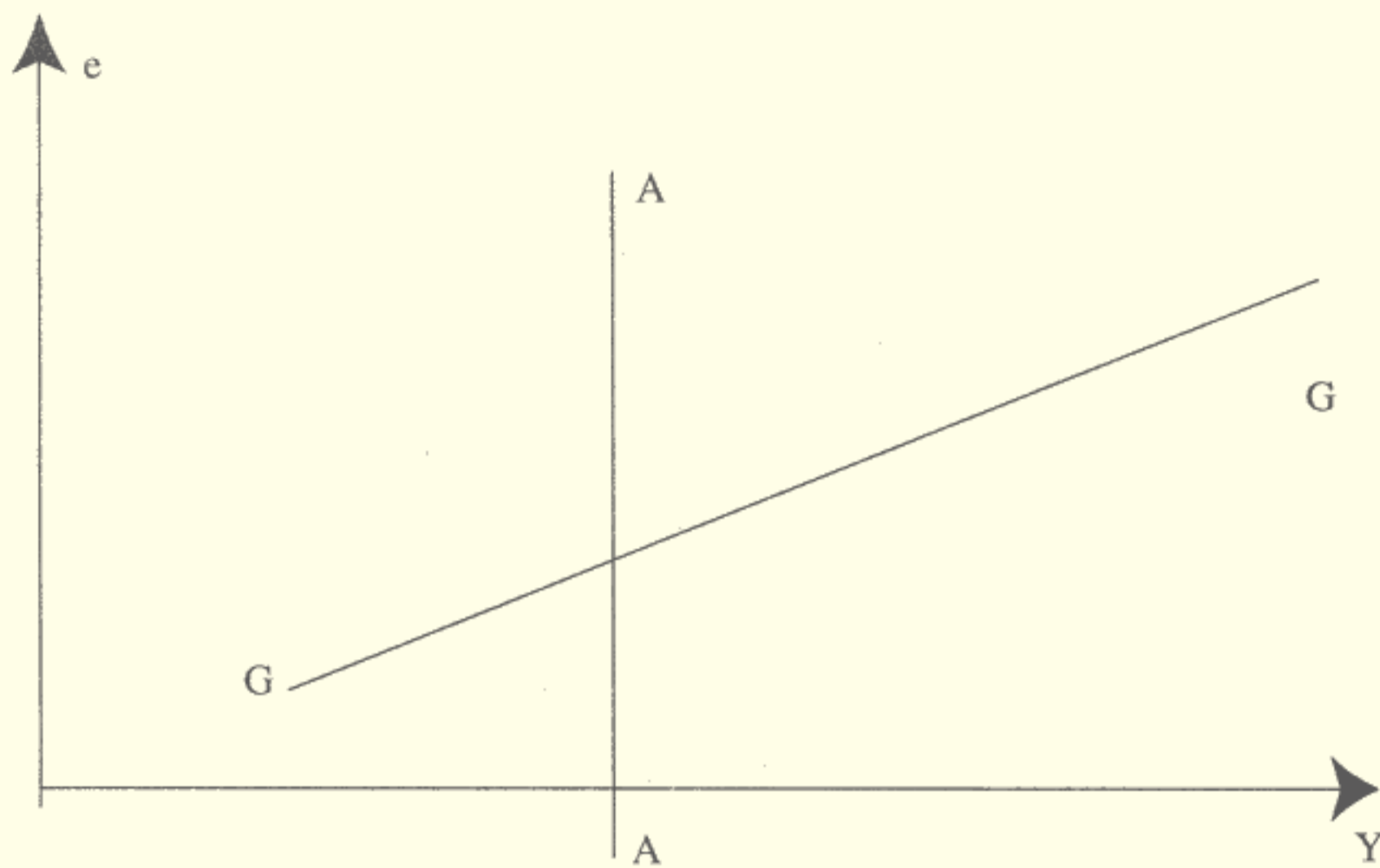
$$M/P = L(Y, i) \quad (10)$$

$$i = i^* \quad (11)$$

with NX , net exports, eP/P , real exchange rate and (11) the arbitrage equation. Figures 3 and 4 represent the dynamics of the model variants.

The line A-A represents all the points at which, given (10), the domestic and foreign interest rates are equal, see figure 3.

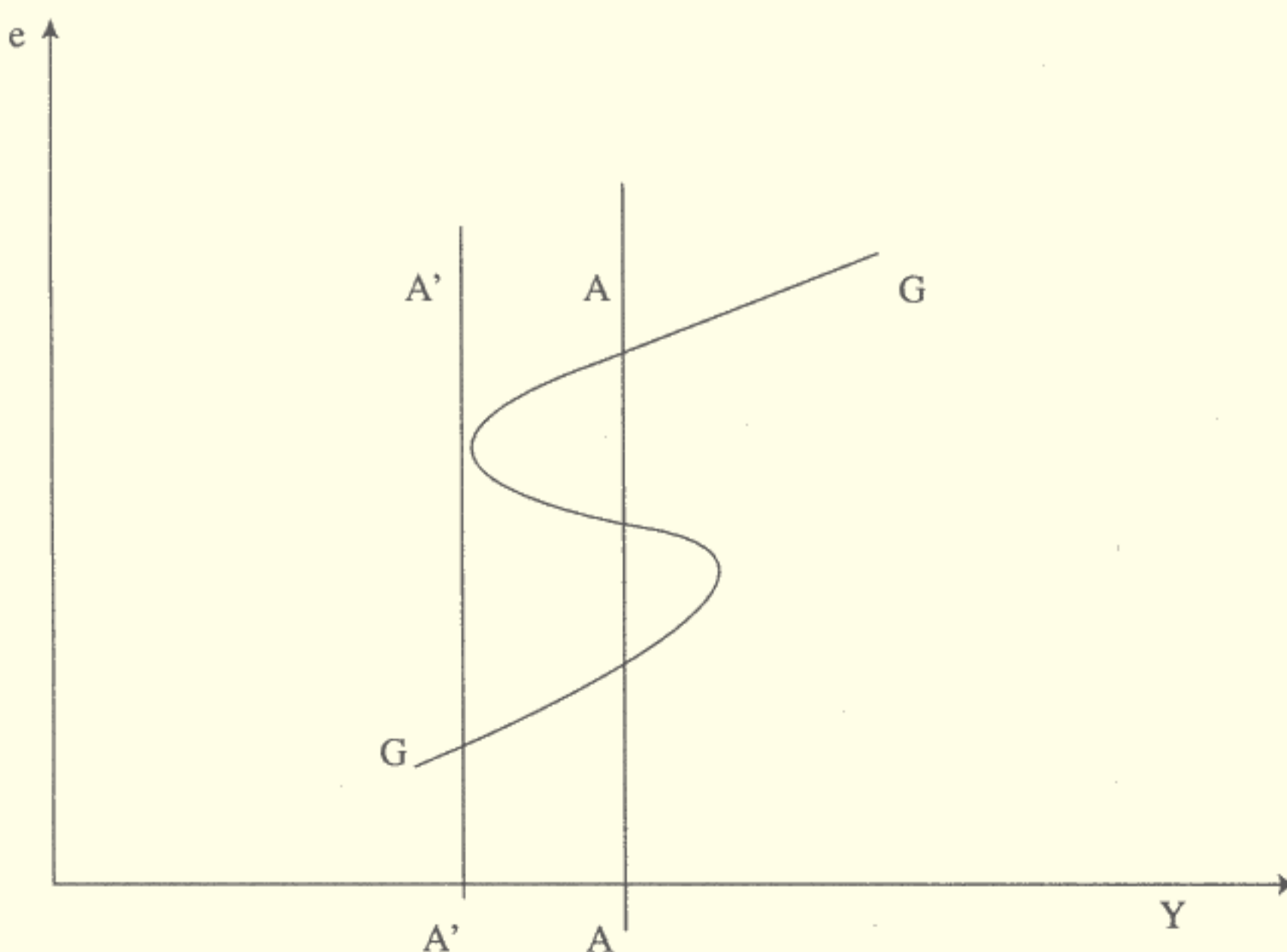
Figure 3
The IS-LM model



The line G-G in figure 3 shows that output is positively influenced by a rising e (depreciation of currency). The modified IS-model reads as follows. With a large fraction of debt (foreign debt) denominated in foreign currency, the networth effect, becomes important with the devaluation of the currency. So we can write (9) as

$$Y = D(Y, i, eP^* / P) + NX(eP^* / P, Y) \quad (9')$$

Figure 4
The IS-LM model with multiple equilibria



There is a nonlinear feedback effect from exchange rates to net worth and demand. This may give rise to the fact that the economy gets stuck at a low level IS-equilibrium entailing a large output loss. It is thus not a quick convergence to a steady state that makes a financially caused downturn a transitory phenomenon but it is rather the switch from high to low level IS-equilibria that seems to cause a protracted crisis. Thus, if the economy is close to the middle point of the A-A and G-G curve in figure 4 (and to the left of A-A), the economy is likely to contract with a sudden depreciation of the currency and may end up in excessive depreciation and low output (see A'-A' curve).

Exchange Rates, Imperfect Capital Markets and Multiple Equilibria

Implicit in Krugman's theory is already the assumption of imperfect capital markets.² The development of the economics of information has made possible the theoretical analysis of credit market imperfections. The main concepts are asymmetric information, moral hazard and adverse selection. Asymmetries of information refers to the borrower-lender relationships. For lenders it is costly to acquire information about the opportunities, characteristics, or actions of borrowers. Financial contacts have to take care of information cost which increases the agency costs. Since risk in credit markets increase the real cost of extending credit reduces it also the efficiency of the process of matching lenders and potential borrowers. These circumstances may have extensive real effects.

The literature of economics of imperfect information has made an attempt to rationalize several characteristics of credit markets such as the form of financial contract, the existence of financial intermediaries, the form of the bankruptcy and the existence of credit rationing. Even though the diverse models in the literature differ in their basic features and predictions, three basic results emerge, provid-

² For details of the role of imperfect capital markets for economic activity and financially caused recessions, see Semmler and Wöhrmann [1999] chs. 3-4.

ing the basis for financial crises. First, external finance is more expensive than internal finance, unless the external finance is fully collateralized. The agency cost of lending is the reason for the higher cost of external finance. Second, given the amount of finance required, the premium on external finance depends inversely on the borrower's net worth. Third, a decrease in the borrower's net worth causing a rise in the premium on external finance reduces spending and investment of the borrower. This result provides the key to the financial crisis. Since adverse shocks to the economy reduce the net worth of borrowers (or through positive shocks net worth increases), the spending and production effects of the initial shock will be propagated and amplified.

Those three results, can be illustrate by employing the model by Kiyotaki and Moore [1995]. Their basic framework can be described as follows. There are two periods in this model: 0 and 1. An entrepreneur operates a technology that uses input in period 0 to produce output in period 1. Moreover, there are two types of input: a fixed factor K (already in place) and a variable input χ_1 . The fixed factor could be a durable input as land. The variable input could be raw materials, labor or firm-specific capital. Finally, at the end of period 1, the entrepreneur can sell the fixed factor at the market price, q_1 , per unit. The variable input depreciates fully in use and its price is normalized to one.

Output in period 1 is $\alpha_1 f(\chi_1)$, where α_1 is a technology parameter and $f(\cdot)$ is increasing and concave. In period 0 the entrepreneur begins with gross cash flow from previous production, $\alpha_0 f(\chi_0)$, and a debt obligation inherited from the past, $r_0 b_0$, where b_0 is past borrowing and r_0 is the gross real interest rate. The link between the entrepreneur purchases of the variable input χ_1 and the borrowing b_1 is given by

$$x_1 = \alpha_0 f(x_0) + b_1 - r_0 b_0 \quad (12)$$

A gross real interest rate of r_1 is imposed to funds borrowed in period 0 and repaid on period 1. The entrepreneur chooses χ_1 and b_1 to maximize period-1 output net of debt repayment. There exists an incentive problem. It is costly

for the lender to seize the entrepreneur's output in case of default. The ownership of the fixed factor is transferred to the lender in case the borrower does not pay his obligations. Therefore, the fixed factor serves as collateral. We can see that the funds provided by the lender in period 0 will be limited by the discounted market value of the fixed factor:

$$b_1 \leq (q_1 / r_1) K \quad (13)$$

There is a collateral-in-advance constraint on spending on the variable input since unsecured lending is not feasible in this model. The incentive constraint is obtained by combining equation (12) and (13):

$$x_1 \leq \alpha_0 f(x_0) + (q_1 / r_1) K - r_0 b_0 \quad (14)$$

where the right hand side of the above equation represents entrepreneur's net worth. This equation tells us that spending on the variable input cannot exceed the entrepreneur's net worth, equal to the sum of cash flow $\alpha_0 f(\chi_0)$ and net discounted assets, $(q_1 / r_1) K - r_0 b_0$. If the entrepreneur's net worth is less than the unconstrained optimal value of x_1 (which satisfies a $\alpha_1 f(\chi_1) = r_1$), then the constraint (14) binds).

This simple framework illustrates the results described earlier. First, when the incentive constraint (14) binds, the shadow value on an additional unit of internal funds is $\alpha_1 f'(\chi_1)$, which exceeds the gross real interest rate prevailing in external capital markets, r_1 . This difference reflects the agency cost of lending. Second, a decrease in the entrepreneur's net worth³ increases the agency premium, and reduces the borrowers spending (for the intermediate input) and production. The financially caused recession can be explained by a shock to the borrower's net worth lead to a down turn of real economy and large output loss.

This incentive constraint (14) also shows different factors that affect the borrower's net worth and therefore the bor-

³ This decrease can be the result of a decline in cash flow or a lower value of the collateralizable asset.

rowers' spending and the levels of production. A decrease in gross cash flows $\alpha_0 f(\chi_0)$, a fall in asset prices q_1 , or an increase in initial debt obligations b_0 reduce net worth. All of them make the constraint binding earlier. If the collateral constraint binds, an increase in r_1 reduces the borrower's spending by a decrease in asset values and therefore in the borrower's net worth. An increase in the interest rate on previous debt, r_0 , also reduces the borrower's spending since it reduces cash flow net of current interest payments $(\alpha_0 f(\chi_0) - r_0 b_0)$.⁴

Miller and Stiglitz [1999] follow the approach by Kiyotaki and Moore [1995] by including exchange rates and debt denominated in foreign currency in a model of imperfect capital markets. This variation of the model gives rise to multiple equilibria.

The Miller and Stiglitz paper concentrates on negative supply-side effects which occur due to balance sheet effects arising from an unexpected devaluation of the currency and the impact on highly-leveraged, fully collateralized firms who have borrowed in foreign currency. A fall in the currency triggers margin calls and consequently a "fire-sale" of collateralized assets; the economy then collapsing to a low level equilibrium and a large output loss.

Formally we can write

$$q_t (k_t - k_{t-1}) + Rb_{t-1} = \alpha k_{t-1} + b_t \quad (15)$$

with q , asset price, b , debt, αk , income and $R = 1 + r$. with r the interest rate. From the above we get

$$b_t = (1 + r)b_{t-1} - (\alpha k_{t-1} - q_t (k_t - k_{t-1})) \quad (16)$$

With χ = the loss arising from the unexpected devaluation of the foreign currency loans we have

⁴ It is important to note that in this example only perfectly secured lending is feasible, so that actual defaults never occur unless there are unexpected shocks to the present value of the borrower (see the subsequent models). However, some models modify this framework allowing for unsecured lending and the possibility of default. This is the case in models known as "costly state verification" (cvc) framework in which the probability of costly auditing by the lender adds to the use of collateral as a way to screen borrowers. This additional mechanism make unsecured lending feasible in equilibrium so that default occur with some positive probability.

$$b_t = (1 + r)b_{t-1} - (\alpha k_{t-1} - q_t (k_t - k_{t-1}) - x) \quad (17)$$

Without the shock χ we have: $b \leq \alpha k/r$. Here again, as in Kiyotaki and Moore, the debt should be smaller than discounted present value of the income stream αk serving as collateral. However, with a shock χ we have:

$$b > \left(\frac{\alpha k - \chi}{r} \right).$$

The latter case arises from a collateral shock (triggered by unexpected devaluations of the currency) leading possibly to a "fire-sale" of collateralized assets and a fall of q whereby the economy is likely to end up in a large output loss and low level equilibrium. Note that, here again not all shocks will drive the economy to a low level equilibrium. Only large shocks accelerated by bad balance sheets will lead to a macro-caused financial and real crises. Miller and Stiglitz estimate the thresholds for those shocks a thirty to forty percent unexpected devaluation of the currency to generate such a systemic crisis.

Exchange Rates, Endogenous Credit Cost and Multiple Equilibria

In the Miller and Stiglitz model the interest rate, the credit cost, per unit of currency borrowed, is fixed. Yet, one of the major issues in modern credit market theory is that credit costs is state dependent. Each agent is likely to face his or her own credit cost. While the main features of the Miller and Stiglitz model are preserved this additional aspect is modeled next.

Credit market imperfections suggest that credit is cost state dependent. In a first view interest rates are perceived indeed as being convex in the agents debt. This has been discussed in [Bhandari, Haque and Turnovsky 1995]. Work on endogenous credit cost can also be found in Bernanke and Gertler [1989], Bernanke, Gertler and Gilchrist [1998]. In their models credit cost depends on net worth of the agents' (households, firms, countries). Net worth is the difference of the agents' own

assets minus liabilities. We follow a similar idea and make the agents credit cost dependent on assets as well as liabilities (debt). The agents liability may depend on the debt denominated in foreign currency and thus on the exchange rate. In addition in our model there is an adjustment cost of capital which prevents capital from being costlessly reallocated. Due to those additional assumption in a credit market model with imperfect capital markets there can be multiple equilibria. Thus for income shocks or changes of the credit cost function there can be different domains of attraction and the economy can, due to shocks, move down from high to low level equilibria exhibiting a large output loss.

Our model starts from the Miller and Stiglitz [1999] model. In the Miller and Stiglitz case there is a discrete time debt accumulation equation:

$$b_t = (1 + r) b_{t-1} - (\alpha k_{t-1} - q_t (k_t - k_{t-1}) - x) \quad (18)$$

where b_t is debt, αk_{t-1} , the income, q_t the price of the investment good (in their case land) and $k_t - k_{t-1}$ the investment (land) and χ = income loss due to unexpected devaluation of the currency.

In our proposed model there are two changes compared to Stiglitz and Miller: first, there is endogenous credit cost. Thus we posit a credit cost $H(k, B)$ instead of rB , above, and second we take as net income

$$\alpha k_{t-1} - q_t (k_t - k_{t-1}) = f(k, j) = k^\alpha - j - \gamma^\beta k^{-\gamma} \quad (19)$$

where $\gamma, \alpha, \beta > 0$. The right hand side of (19) represents income generated from a production function minus investment (including an adjustment cost for capital). More specifically, our model reads as follows. We consider a continuous time model and for net income

$$f_t = \alpha k_{t-1} - q_t (k_t - k_{t-1})$$

we take

$$f(k, j) = k^\alpha - j - j^\beta k^{-\gamma} \quad (20)$$

with the evolution of capital stock given by

$$\dot{k} = j - \sigma k, k(0) = k_0 \quad (21)$$

With endogenous credit cost $H(k, B)$ we have the evolution of debt

$$\dot{B} = H(k, B) - f(k, j) \quad (22)$$

where $H(k, B)$ is the above mentioned endogenous credit cost. We define creditworthiness, $B^*(k)$, the maximum amount that the economic agent (household, firm, government or country) can borrow given the initial conditions $k(0) = k_0, B(0) = B_0$.

Note that if the interest rate $r = \frac{H(k, B)}{B}$ is constant, as in the Miller and Stiglitz case, then, as is easy to see, $B^*(k)$ is the present value of the income stream generated by k : (subtracting the initial debt $B(0)$)

$$B^*(k) = \text{Max}_j \int_0^\infty e^{-rt} f(k, j) dt - B(0) \quad (23)$$

$$\dot{k} = j - \sigma k, t \geq 0, k = k(0) \quad (24)$$

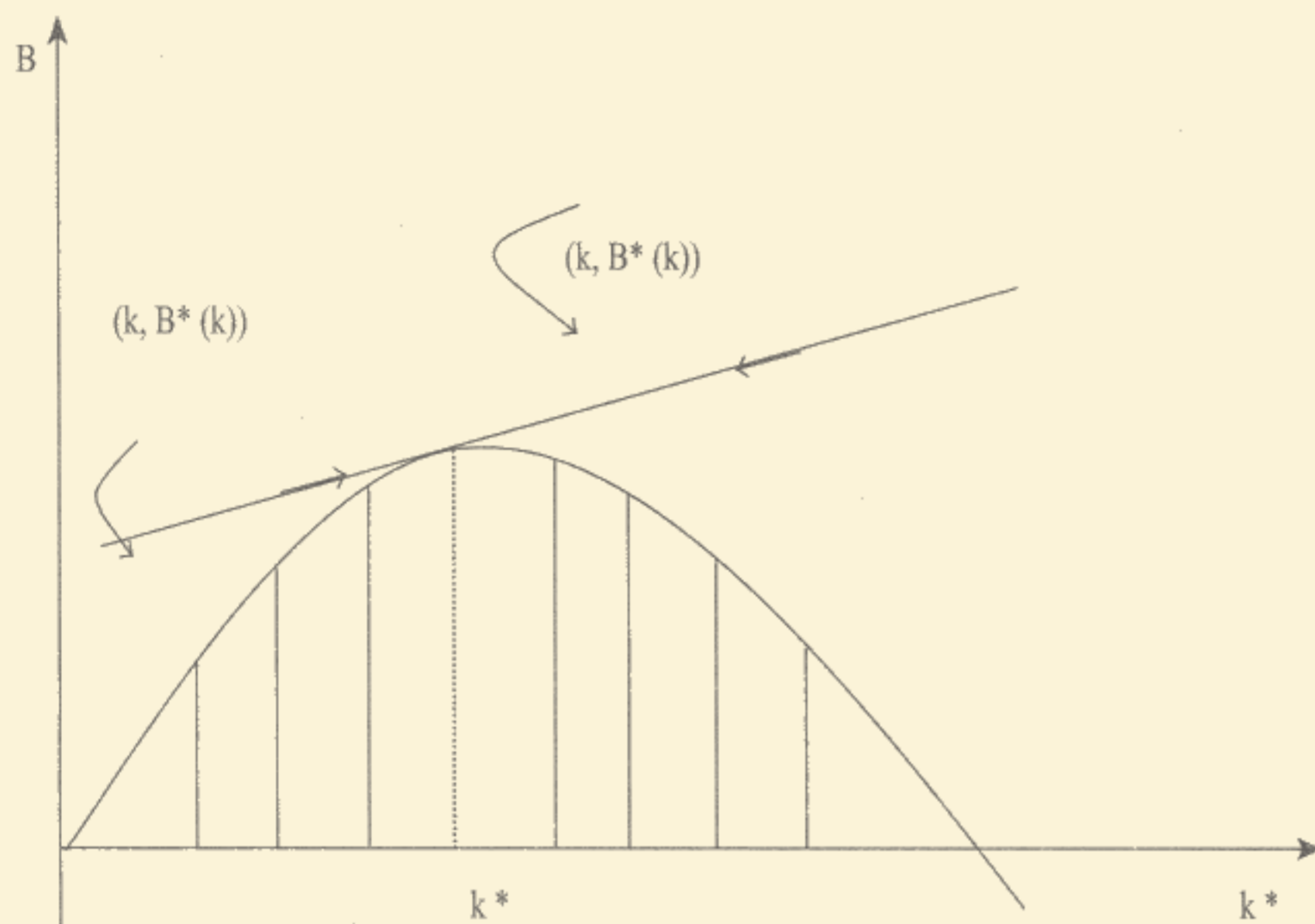
$$\dot{B} = rB - f(k, j), B(0) = B_0 \quad (25)$$

In Semmler and Sieveking [1999] the more general case where r is not a constant is considered. Then not only the relation of the present value to creditworthiness but also the notion of present value itself become difficult to treat. Note that the endogenous credit cost $H(k, B)$ is determined by creditworthiness $B^*(k)$ and on the other hand, the maximum amount an agent can borrow depends on the credit cost. This is the reason why commonly used present value computations (through the Hamiltonian) is not feasible. Semmler and Sieveking [1999] develop a special technique to solve this problem.

Moreover, public debt moves down $B^*(k)$ down and exchange rate shocks (depreciation of the currency) decrease net income and possibly increase $H(k, B)$. Due to the

assumed nonlinear relationship in the model (nonlinear cost of capital adjustment and the nonlinear credit cost function) there can be multiple steady states. The possibility of a unique steady state is illustrated in figure 5.

Figure 5
Model with endogenous credit cost
and unique equilibrium



Below the line $(k, B^*(k))$, moving from both sides into the steady state k^* , the agent is creditworthy because the value of debt is lower than the present value from the agent's action. Above that line the agent will be bankrupt.

Figure 6
Model with endogenous credit cost
and multiple equilibria

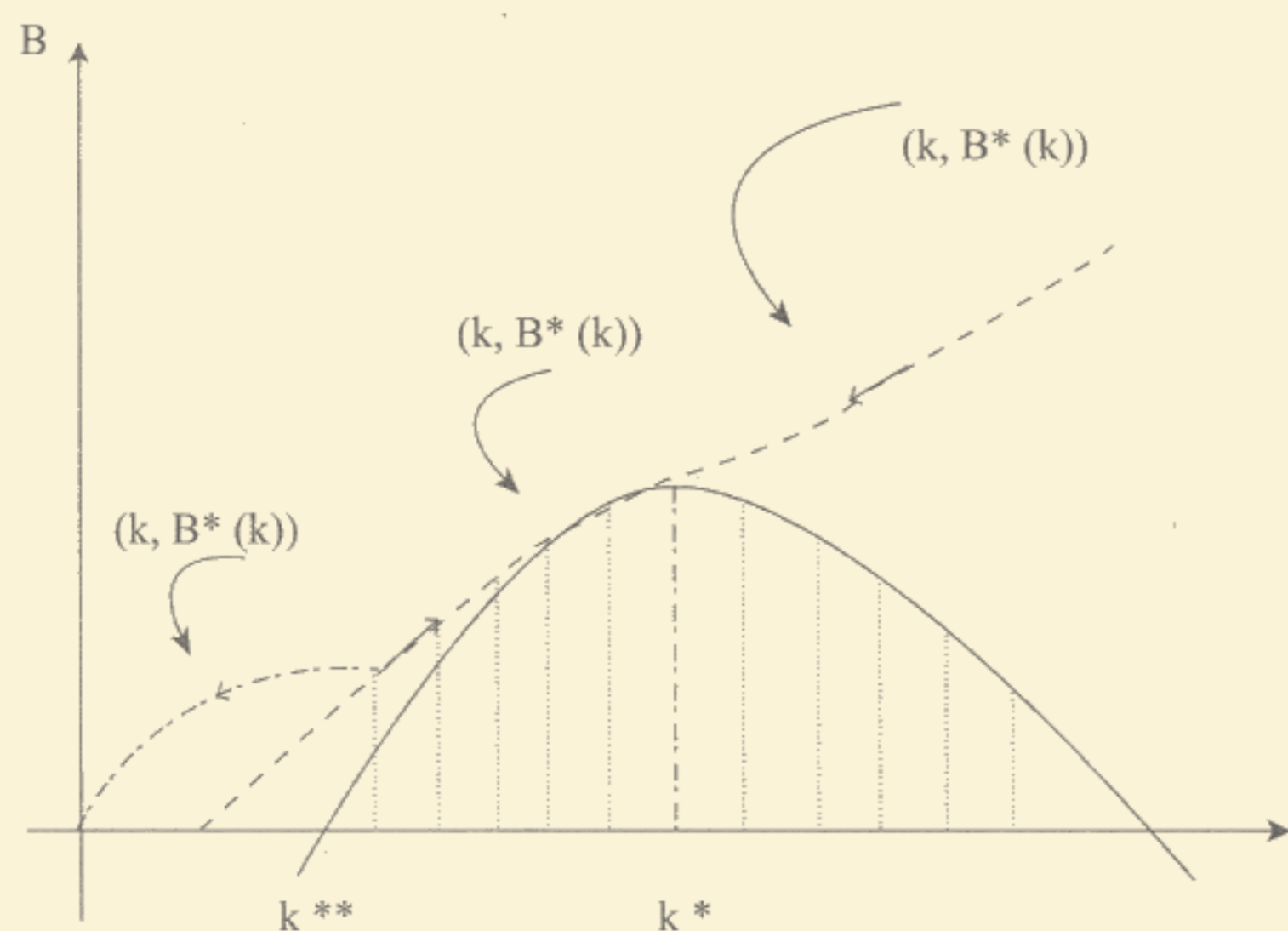


Figure 6 shows the case when there are multiple steady state equilibria. Again, below the dotted line the agent will be solvent and above that line bankruptcy will arise. Note that the slope $(k, B^*(k))$ of the line depends on $H(k, B)$, the credit cost function. A large shock to the net income function, a large shock to the exchange rate, an increase to the initial debt or a change of the credit cost function $H(k, B)$ which makes credit cost rising, will either render the agent, –in our case the country– insolvent or make the low level equilibrium (the one with large output loss) an attractor. Numerical examples of those outcomes and further discussions are provided in Semmler and Sieveking [1999].

Conclusions

This paper studies stylized facts and the basic mechanisms of financially caused recessions. As we have shown it is likely to be the connection of weak balance sheets (of households, firms, financial intermediaries, governments and countries) and large exchange rate shocks that lead to positive feedback mechanisms and thus to real crisis and large output losses. This in particular is a basic mechanism if there exists in the country large foreign debt denominated in foreign currency. Moreover, as we have shown, if credit cost is endogenous (state dependent) such positive feedback mechanisms, leading to low level equilibria are strongly enforced.

Appendix: Testing Sustainability of Foreign Debt

Following Flood and Garber [1980] and Hamilton and Flaven [1986] a NLLS estimate for the sustainability of external debt can be undertaken. This is pursued in Semmler and Sieveking [1999].

As time series data one needs trade account and the net foreign assets. A discrete time version of external debt dynamics, with initial debt B_0 , can be written as

$$B_t = (1 + r_{t-1}) B_{t-1} - TA_t \quad (26)$$

where TA_t is the trade account and B_{t-1} the external debt last period. Recursive substitution forward (with constant interest rate) leads to

$$B_t = \sum_{i=t+1}^N \frac{TA_i}{(1+r)^{i-t}} + \frac{(1+r)^t B_N}{(1+r)^N} \quad (27)$$

In the equ. (27) the second term must go to zero if the intertemporal budget constraint is supposed hold. Then:

$$B_t = E_t \sum_{i=t+1}^{\infty} \frac{TA_i}{(1+r)^{i-t}} \quad (28)$$

Equivalent to requiring that equ. (28) must be fulfilled is that the following condition holds

$$E_t \lim_{N \rightarrow \infty} \frac{B_N}{(1+r)^N} = 0 \quad (29)$$

The equation is the usual transversality condition or non-ponzi game condition.

If the foreign debt is constrained not to exceed a constant, A_0 , on the right hand side of (27), we then have

$$B_t = E_t \sum_{i=t+1}^{\infty} \frac{TA_i}{(1+r)^{i-t}} + A_0 (1+r)^t \quad (30)$$

The NLLS test proposed by Flood and Garber [1980] and Hamilton and Flaven [1986] can be modified for our case. It reads:

$$TA_t = b_1 + b_2 TA_{t-1} + b_3 TA_{t-2} + b_4 TA_{t-3} + \varepsilon_{2t} \quad (31)$$

$$B_t = b_5 (1+r)^t + b_6 + \frac{(b_2 b + b_3 b^2 + b_4 b^3) TA_t}{(1 - b_2 b - b_3 b^2 - b_4 b^3)} \quad (32)$$

$$+ \frac{(b_3 b + b_4 b^2) TA_{t-1}}{(1 - b_2 b - b_3 b^2 - b_4 b^3)} + \frac{(b_4 b) TA_{t-2}}{(1 - b_2 b - b_3 b^2 - b_4 b^3)} + \varepsilon_{1t}$$

An estimation with time series data for the European Monetary Union is performed in Semmler and Sieveking

[1999]. This approach can, however, also be used to estimate debt sustainability for any other country or region.

References

- Adelzadeh, A. and W. Semmler [1998], "Open Economy Macrodynamics with Least Square Learning of the Exchange Rate", New School for Social Research, mimeo.
- Burnside, C., M. Eichenbaum and S. Rebelo [1999], "Hedging and Financial Fragility in Exchange Rate Regimes", paper presented at the CEPR Conference in Perugia, September.
- Bhandary, J.S., N.U. Haque and S.J. Turnovsky [1990], "Growth, External Debt, and Sovereign Risk in a Small Open Economy", IMF Staff Papers, vol. 37, no. 2, International Monetary Fund.
- Chiarella, C., P. Flaschel, G. Groh and W. Semmler [2000], *Disequilibrium, Growth and Labor Market Dynamics*, Heidelberg/New York, Springer Verlag.
- Dornbusch, R. [1976], "Expectations and Exchange Rate Dynamics", *Journal of Political Economy* 84 (December): 1161-1176.
- Flood, R.P. and P.M. Garber [1980], "Market Fundamentals Versus Price Level Bubbles: The First Tests", *Journal of Political Economy* 88: 745-770.
- Gray and Turnovsky [1979], *International Economic Review*, vol. 20, no. 3.
- Hamilton, J. and M.A. Flavin [1986], "On the Limitations of Government Borrowing: A Framework for Empirical Testing", *American Economic Review* 76(4): 808-819.
- Kiyotaki, N. and J. Moore [1991], "Credit Cycles", *Journal of Political Economy*, vol 105, April: 211-248
- Krugman, P. [1999a], "Balance Sheets, the Transfer Problem and Financial Crises", MIT, mimeo.
- ——— [1999b], "Comments on the Transfer Problem", MIT, mimeo.
- Milesi-Ferretti, G.M. and A. Razin [1997], "Sharp Reduction in Current Account Deficits: An Empirical Analysis". NBER working paper no. 6310.
- ——— and A. Razin [1996], "Current Account Sustainability: Selected East Asian and Latin American Experiences". NBER working paper, no. 5791.

- Mishkin, F. [1998], "International Capital Movements, Financial Volatility and Financial Instability", NBER working paper, no. 6390.
- Miller, M. and J. Stiglitz [1999], "Bankruptcy Protection against Macroeconomic Shocks", The World Bank, mimeo.
- Semmler, W. and A. Greiner [1999], "Monetary Policy and Non-Uniqueness of Unemployment Steady States", paper prepared for the CEPR conference on Expectations, Economic Theory and Economic Policy, Perugia, September.
- Semmler, W. and M. Sieveking [1999], "Credit Risk and Sustainable Debt: A Model and Estimation for Euroland", Dept. of Economics, University of Bielefeld, mimeo.
- Semmler, W. and M. Sieveking [2000], "Critical Debt and Debt Dynamics", *Journal of Economic Dynamics and Control*, vol. 24:1121-1144.
- Semmler, W. and P. Wöhrmann [1999], "Lectures on the Financial Market and Economic Activity", New School University and University of Bielefeld, mimeo.