REVISITING PURCHASING POWER PARITY IN EMERGING-7 COUNTRIES:
A POWERFUL UNIT ROOT TEST

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ABSTRACT

This paper introduces a newly developed unit root test procedure named the Fourier Quantile AESTAR (FAESTAR-QKS) test that allows nonlinearity and structural changes. The FAESTAR-QKS unit root test is mainly based on the quantile approach and provides more powerful results since it is robust toward non-normal errors. Then, we test the Purchasing Power Parity hypothesis (PPP) [or the mean-reverting properties of real exchange rates] in emerging seven (E7) countries (Brazil, China, India, Indonesia, Mexico, Russia, and Turkey) from 1995:1 to 2023:6 by using a novel FAESTAR-QKS test procedure. The results show that the FAESTAR-QKS unit root test provides more evidence on the validity of PPP than the traditional unit root test. Accordingly, the PPP hypothesis is valid in all E7 countries except for Turkey in the long run.

Keywords: PPP, quantile unit root, E7 countries.
JEL **Classification:** C22, F31, F41.

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LA PARIDAD DEL PODER ADQUISITIVO EN LOS PAÍSES EMERGENTES (E7): UNA PRUEBA DE RAÍZ UNITARIA ROBUSTA

RESUMEN

Este artículo estima un método recientemente desarrollado llamado Fourier Quantile AESTAR (FAESTAR-QKS). Es una prueba de raíz unitaria que permite la no linealidad y los cambios estructurales. La FAESTAR-QKS se basa principalmente en un enfoque de cuantiles y dado que es más robusta frente a errores que no presentan una distribución normal proporciona resultados más eficaces. Con base en esta metodología probamos la hipótesis de la paridad de poder de compra (PPC) [o las propiedades de reversión de los tipos de cambio reales] en los países emergentes (E7), a saber: Brasil, China, India, Indonesia, México, Rusia y Turquía para el periodo 1995:1 a 2023:6. Los resultados muestran que la prueba de raíz unitaria FAESTAR-QKS proporciona evidencia más robusta sobre la validez de la PPC que la prueba de raíz unitaria convencional. Finalmente comprobamos que, a excepción de Turquía, la hipótesis de la PPC es válida en el largo plazo para la mayoría de los países analizados.
Palabras clave: PPC, raíz unitaria del cuantil, países E7.
Clasificación JEL: C22, F31, F41.

1. INTRODUCTION

Since the breakdown of the Bretton Woods System in the early 1970s, researchers have conducted an ongoing and lively debate regarding the validity of Purchasing Power Parity (PPP) and the stationarity of Real Exchange Rates (RER). In particular, the gradual advances in new econometric testing methods (as well as the expansion in data range) have contributed to the formation of an important body of literature specific to the theory of PPP.

The Swedish economist Gustav Cassel (1916; 1918) proposed this theory because currency values faced inflation during World War I, and calculating their real values was critical to re-establishing international trade (Mike and Kızılkaya, 2019). However, the theory has a different intellectual origin. Dornbusch (1985) credited the Salamanca School in sixteenth-century Spain and the writings of Gerrard de Malynes in 1601 in England. Officer (1976) attributed it to the work of British economists on the floating pound during the Bank Restriction Period (1797-1821). Frenkel (1978), however, stated that PPP is based on the works of Wheatley and Ricardo. While much discussion about the general validity of PPP has been developed, the theory plays a crucial role in explaining the important factors behind exchange rate movements.

PPP asserts that the nominal exchange rate between two currencies must equal the relative price of the countries in question. The basic idea behind PPP is that commodity prices expressed in a common currency in different countries will eventually equalize after arbitrage activities (Doğanlar, Mike and Kızılkaya, 2021). This theory is also crucial for the foreign exchange market to determine the purchasing power of currencies. Accordingly, PPP predicts that a fall (or an increase) in the purchasing power of domestic currency will be associated with proportional currency depreciation (appreciation) in the foreign exchange market (Krugman, Obstfeld, and Melitz, 2018).

Although PPP makes important contributions to exchange rate determination theories, it has been criticized by many economists since it was introduced to the literature (Boundi-Chraki and Mateo Tomé, 2022; Vo and Vo, 2023). For example, Keynes (1923; 1930) stated that the PPP hypothesis could be accepted as a "truism" and various transaction costs (transport charges, import and export taxes, and tariffs, etc.) may invalidate this hypothesis. Similarly, Dornbusch (1985) stated that the existence of heterogeneous baskets of goods among countries contradicts the law of one price and PPP. Finally, Taylor and Taylor (2004) drew attention to the existence of non-tradable goods, the degree of weighting of similar goods in aggregate price indices, and differences in labor and property costs across countries. They also pointed out that these factors pose significant challenges at both the theoretical and empirical levels.

All these criticisms have led to the emergence of two puzzles for the PPP. Taylor (2006) establishes that the first PPP puzzle lies in the absence of robust empirical evidence supporting the long-run relationship. Rogoff (1996) identifies the second PPP puzzle, which is the inconsistency between the short-term volatility of the real exchange rate and its slow adjustment to PPP in the long run. In this context, the modern literature refers to the emphasis made by Heckscher (1916) that the adjustment of real exchange rates to PPP is likely to be nonlinear due to transaction costs and international arbitrage. In particular, Dumas (1992) provided an important insight into the nature of deviations from the PPP. Accordingly, the deviations from PPP follow a nonlinear process, and the adjustment rate towards equilibrium varies according to the size of the deviation from PPP (Michael, Nobay and Peel, 1997).

These objections notwithstanding, however, it is widely argued that the PPP theory serves as an anchor in determining the long-run equilibrium exchange rate (Rogoff, 1996; Sarno and Taylor, 2002; Taylor and Taylor, 2004). In this context, PPP theory mainly has two interpretations: The absolute and relative versions. The absolute version of PPP (or strong form), based on the law of one price, states that the nominal exchange rate should be equal to the price indices ratio of the countries. On the other hand, relative PPP (or weak form) reveals that the percentage changes in the nominal exchange rate should be equal to the percentage changes in the domestic and foreign price levels. Since prices are relatively inflexible in response to changes in the nominal exchange rate in the short run, both approaches are considered long-run theories for determining exchange rates (Cuestas and Regis, 2013). However, some forces are capable of bringing the exchange rate back to its equilibrium values in the long run (He and Chang, 2013).

The long-run validity of PPP is analyzed in these two forms (Dornbusch, 1985). The strong form of PPP is tested with unit root analyses for the RER series, and therefore, whether the RER series shows the mean-reverting properties in the long run is investigated. If RER s contain a unit root, shocks are permanent, and thus PPP becomes invalid. On the other hand, the weak form of PPP is tested using cointegration analyses to investigate the long-run relationship between nominal exchange rate and relative price levels. If there is a cointegration relationship between the series, it can be concluded that the PPP is valid in the long run (Doğanlar, Mike and Kızılkaya, 2021).

The validity of the PPP theory is analyzed with the different unit root procedures since the 1970s. Earlier studies investigating the validity of PPP often employed traditional unit root tests, yielding mixed results (Sabaté, Gadea and Serrano, 2003). One potential explanation for these inconclusive findings lies in the limited power of traditional tests, particularly when structural breaks exist in the RER data. These breaks, often corresponding to significant economic or political events (*e.g.*, financial crises, policy changes), can significantly alter the relationship between exchange rates and prices. Ignoring such breaks can lead to spurious results and invalid conclusions about PPP validity due to misinterpretations of the data. Recent studies have adopted unit root tests that explicitly account for structural changes to address this limitation. Such as the Zivot-Andrews (1992), Lumsdaine-Papell (1997), and Perron (1997) unit root tests, which utilize dummy variables to detect breaks. However, these tests primarily capture abrupt and permanent changes, often associated with currency crises. In such cases, a qualified version of PPP, termed "quasi- PPP," holds instead of absolute PPP (Papell and Prodan, 2006). Conversely, temporary structural changes would support the long-run validity of the traditional PPP hypothesis. Building upon this understanding, recent studies have explored the PPP validity employing unit root tests incorporating Fourier functions with integer frequencies. This approach ensures that the identified breaks are temporary, as the Fourier function's starting and ending values coincide (Christopoulos and León-Ledesma, 2010). Christopoulos and León -Ledesma (2010) developed a unit root test that allows both structural breaks and nonlinearities to investigate the mean-reverting properties of the RER series. However, Bahmani-Oskooee, Chang and Ranjbar (2017) emphasized that both linear and nonlinear unit root tests commonly focus on the average behavior of the RER series, which ignores the various sizes and signs of the shocks. These issues are resolved by the quantile unit root test which was introduced by Koenker and Xiao (2004). Yet, Bahmani-Oskooee, Chang and Ranjbar (2017) criticized this method because it does not consider the structural breaks. To overcome these issues, we employ unit root tests with Fourier functions to assess the long-run validity of PPP, avoiding potential biases associated with quasi- PPP arising from permanent structural breaks.

This article introduces a newly developed unit root test procedure named the Fourier quantile AESTAR (FAESTAR-QKS) test that allows nonlinearity and structural changes. Bahmani-Oskooee *et al.* (2020) have already suggested a nonlinear quantile unit root test based on an Exponential Smooth Transition Autoregressive (ESTAR) model. In this study, instead of the ESTAR model, we consider the asymmetric ESTAR model. The FAESTAR-QKS unit root test, based on the quantile approach and the quantile-based unit root tests, has some advantages. For example, the quantile unit root methodology provides a powerful test since it is robust toward non-normal errors. Besides, the quantile unit approach allows us to test whether a unit root exists at the quantile interval and each single quantile level (Ma, Li and Park, 2017).

Within this context, this paper examines the PPP hypothesis for emerging seven (E7) countries (Brazil, China, India, Indonesia, Mexico, Russia, and Turkey) from 1995:1 to 2023:6 using the Fourier quantile AESTAR unit root test. This paper contributes to the existing literature on two fronts. First, it suggests a new quantile unit root test considering both nonlinearity and multiple smooth breaks and provides more reliable results than traditional, nonlinear, and Fourier-type unit root tests. Second, it is one of the few studies that test the PPP hypothesis for E7 countries.

The rest of this paper is organized as follows. Section 2 gives a summary of the related literature. Section 3 includes the data and methodological approach of the paper. Section 4 presents the empirical results. Finally, Section 5 concludes.

2. LITERATURE REVIEW

The PPP hypothesis has been widely tested in the literature with mixed results. The PPP hypothesis is mainly tested by employing unit root tests to determine whether RERs are stationary. The RER, which combines the nominal rate with relative prices, must converge to its mean in the long run. In other words, the RER series must be stationary or should not have any unit root, which indicates that the PPP hypothesis is valid (Bahmani-Oskooee *et al.*, 2018). Early studies adopted this approach, and from the mid to late 1980s onward, they utilized a variant of the Augmented Dickey-Fuller (ADF) test (*e.g.*, Sarno and Taylor, 2002; Taylor, 2006). Earlier studies on the PPP hypothesis also used other conventional unit root tests for different samples, such as the Phillips-Perron (PP), and the Kwiatkowski, Phillips, Schmidt, and Shin (KPSS). Besides, some other studies (*e.g.*, Emirmahmutoglu and Omay, 2014; Hepsag, 2021) have used different advanced unit root tests to examine the PPP hypothesis. Boundi-Chrak and Mateo Tomé (2022) test the PPP hypothesis using traditional and nonlinear unit root tests for Organisation for Economic Co-operation and Development (OECD) countries. The results of traditional and nonlinear unit root tests do not verify the theory. Using newly developed unit root tests, which are classified into four versions of the PPP, according to whether the exchange rate process is level (trend) stationary with temporary (permanent) structural break(s), Xie, Chen and Hsieh (2021) test the PPP hypothesis for 23 OECD countries and the euro area, and the results vary in terms of these four versions.

The second group of studies applied the cointegration methodology, initially developed by Engle and Granger (1987), to test the PPP hypothesis. While early studies (Taylor, 1988; Kim, 1990; Kugler and Lenz, 1993) used the cointegration test to test the PPP hypothesis, they also reported mixed results. Early studies using cointegration methodology present some stylized facts. First, the PPP hypothesis is mostly valid when the WPI (Wholesale Price Index) is utilized instead of the CPI (Consumer Price Index) and, even more so, when the Gross Domestic Product (GDP) deflator is used, more evidence in support of PPP is proposed (Sarno and Taylor, 2002).

A considerable amount of literature has recently employed quantile-based unit root tests to test the PPP hypothesis. Quantile unit root tests are robust to different types of error distributions, particularly heavy-tailed distributions, which is a significant characteristic of many economic series (see Bahmani-Oskooee *et al.*, 2018). Bahmani-Oskooee, Chang and Ranjbar (2017) introduce a new unit root test that combines the quantile unit root test with Fourier expansion. The results of the Fourier quantile unit root test show that the PPP hypothesis is valid for most of the 23 OECD countries. Using the nonlinear quantile unit root test developed by Li and Park (2018), Bahmani-Oskooee *et al.* (2018) show that the PPP hypothesis holds for 15 out of 29 African countries. Bahmani-Oskooee *et al.* (2020) introduce a new Fourier nonlinear quantile unit root test, showing that the PPP hypothesis holds in 21 out of 29 African countries. Using the Fourier quantile unit root test, Doğanlar, Kızılkaya and Mike (2020) show the PPP hypothesis between Turkey and China, the Euro Area, Russia, the UK, and the US. The results indicate that the long-run PPP holds for all countries. Doğanlar, Mike and Kızılkaya (2021) also confirm the validity of the PPP hypothesis in 8 developed, 11 emerging, and 7 frontier market economies using the Fourier quantile unit root test. Bahramian and Saliminezhad (2021) conclude that the PPP hypothesis holds for four ASEAN-5 countries. She *et al.* (2021) test the PPP hypothesis using Fourier unit root tests for Pakistan against 21 trading partners and show that the PPP hypothesis mostly holds. Nazlioglu, Altuntas and Kilic (2022) show that the PPP hypothesis is valid for 26 out of 27 emerging markets using the Fourier nonlinear quantile unit root test.

3. METHODOLOGY

To test the PPP hypothesis in E7 countries, we suggest a new quantile unit root test that considers multiple smooth breaks. We apply a two-step strategy by following Christopoulos and León-Ledesma (2010). In the first step, we estimate the next model:

$$RER_t = \alpha_0 + \alpha_1 \sin\left(2\pi kt/T\right) + \alpha_2 \cos\left(2\pi kt/T\right) + \xi_t \qquad [1]$$

Where RER_t indicates the real exchange rates, k shows a particular frequency, t and T indicate a trend term and sample size, respectively. To determine the optimal value k, we estimate equation [1] for all values in the interval [1, 2, …, 5] and choose the k that minimizes the sum of squared errors. Next, we obtain the residuals of equation [1] as follows:

$$\hat{\xi}_t = RER - \hat{\alpha}_0 - \hat{\alpha}_1 \sin\left(2\pi k^* t/T\right) - \hat{\alpha}_2 \cos\left(2\pi k^* t/T\right) \qquad [2]$$

In the second step, we test for a unit root in these residuals that equal the RER_t series, which are free from the effect of multiple smooth breaks. For this purpose, we employ an Asymmetric Exponential Smooth Transition Autoregressive Model (AESTAR), which was developed by Sollis (2009), and a nonlinear model that uses both an exponential function and a logistic function by assuming the transition variable is $\hat{\xi}_{t-1}$ as follows:

$$\Delta\hat{\xi}_t = G_t\left(\gamma_1, \hat{\xi}_{t-1}\right)\left\{S_t\left(\gamma_2, \hat{\xi}_{t-1}\right)\rho_1 + \left(1 - S_t\left(\gamma_2, \hat{\xi}_{t-1}\right)\right)\rho_2\right\}\hat{\xi}_{t-1} + \sum_{i=1}^{k}\beta_i \Delta\hat{\xi}_{t-i} + \varepsilon_t \qquad [3]$$

Where $$G_t\left(\gamma_1, \hat{\xi}_{t-1}\right) = 1 - \exp\left(-\gamma_1\left(\hat{\xi}_{t-1}^2\right)\right), \gamma_1 \geq 0$$

$$S_t\left(\gamma_2, \hat{\xi}_{t-1}\right) = \left[1 + \exp\left(-\gamma_2 \hat{\xi}_{t-1}\right)\right]^{-1}, \gamma_2 \geq 0$$

While the former shows the exponential function, the latter is the logistic function. Replacing the exponential function with a first-order

Taylor approximation around $\gamma_1 = 0$ and the logistic function with a first-order Taylor approximation around $\gamma_2 = 0$ gives the following model:

$$\Delta\hat{\xi}_t = \theta_1\hat{\xi}_{t-1}^3 + \theta_2\hat{\xi}_{t-1}^4 + e_t \quad [4]$$

To remedy the possible autocorrelation, the lags of the dependent variable could be included in the equation. The null of unit root can be tested via H_0: $\theta_1 = \theta_2 = 0$. In the case of the rejection of the null of unit root, a F-test to examine the significance of the trigonometric terms ($\alpha_1 = \alpha_2 = 0$) can be used. The rejection of the null shows multiple smooth breaks in the data generation process.

We can employ the following nonlinear quantile autoregression model to test the null of a unit root within each quantile of the RER series by following Bahmani-Oskooee *et al.* (2020):

$$Q_{\Delta\hat{\xi}_t}\left(\tau|Z\right) = \theta_1(\tau)\hat{\xi}_{t-1}^3 + \theta_2(\tau)\hat{\xi}_{t-1}^4 + u_t \quad [5]$$

Here $\boldsymbol{Q}_{\Delta\hat{\xi}_t}\left(\tau|\boldsymbol{Z}\right)$ denotes the τ-th quantile of $\Delta\hat{\xi}$ conditional on Z, where Z shows all covariates in equation [5]. We can employ the following Wald test statistic to test the null hypothesis of a unit root at different quantiles [H_0: $\theta_1 = \theta_2 = 0$, $\forall\tau\in(0,1)$]:

$$FQAESTAR(\tau) = \left[T\left(\hat{\theta}_i(\tau)\right)'\left(\Omega(\tau)\right)^{-1}\left(\hat{\theta}_i(\tau)\right)\right]\Big/\tau(1-\tau) \quad [6]$$

Where $\hat{\theta}_i(\tau)$ is the vector of estimated coefficients of τth quantile, $\hat{\Omega}(\tau)$ indicates the consistence estimator of variance-covariance matrix of the $\hat{\theta}_i(\tau)$. The null can be tested over a range of quantiles instead of a selected quantile using the Kolmogorov-Smirnov (QKS) test statistics of Koenker and Xiao (2004).

$$FAESTAR-QKS = \sup_{\tau\in[0.1,0.9]}\left|FQAESTAR(\tau)\right| \quad [7]$$

By following the suggestion of Bahmani-Oskoee, Chang and Ranjbar (2017), we employ bootstrap simulations to compute necessary critical values since asymptotic distributions of test statistics are non-standard.

4. DATA AND EMPIRICAL RESULTS

We employ monthly data for E7 countries, Brazil, China, India, Indonesia, Mexico, Russia, and Turkey. We compute the RER series with the following equation: $RER_t = NER_t + P^* - P$, where NER_t is the nominal exchange rates (the amount of national currency unit per US dollars). P^* and P are the foreign consumer price index (US) and domestic consumer price index (E7 countries), respectively. All series are obtained from the International Monetary Fund's International Financial Statistics database and expressed in their natural logarithms. We first present the descriptive statistics and the data range of the RERs in Table 1.

Table 1 shows that Indonesia has the highest mean among the seven countries, while Turkey has the lowest. The order of the countries based on the median is the same as that based on the mean, with Indonesia having the highest median (9.303) and Turkey having the lowest median (0.833). Russia has the highest standard deviation (0.283), indicating the highest variability among its values, while China has the lowest standard deviation (0.128), marking the least variability among its values. Indonesia

Table 1. Descriptive statistics

Countries	Mean	Median	Std. Dev.	Skewness	Kurtosis	Jarque-Bera (p-value)	Data Range
Brazil	0.898	0.880	0.282	0.231	1.969	18.192 (0.000)	Jan 1995-Jun 2023
China	1.960	1.950	0.128	0.023	1.564	29.411 (0.000)	Jan 1995-Jun 2023
India	3.993	3.970	0.141	0.152	1.559	30.789 (0.000)	Jan 1995-May 2023
Indonesia	9.331	9.303	0.222	1.163	5.992	204.726 (0.000)	Jan 1995-Jun 2023
Mexico	2.617	2.579	0.144	0.449	2.100	23.035 (0.000)	Jan 1995-Jun 2023
Russia	3.736	3.713	0.283	0.402	2.447	12.985 (0.002)	Jan 1995-Mar 2022
Turkey	0.805	0.833	0.282	0.309	2.403	10.481 (0.005)	Jan 1995-Apr 2023

Note: * shows the significance at the 1% level.

has the highest skewness (1.163), showing a highly positively skewed distribution (right tail), while China has the lowest skewness (0.023), indicating an almost symmetrical distribution. Indonesia has the highest kurtosis (5.99), demonstrating a distribution with heavy tails and more outliers. India has the lowest kurtosis (1.559), indicating a distribution with light tails and fewer outliers.

The Jarque-Bera test statistics in Table 1 indicate that all RERs of all countries are distributed as non-normal, which justifies the quantile approach since Koenker and Xiao (2004) noted that the unit root tests based on the quantile regression are more powerful than the traditional unit root tests in the case of non-normally distributed series. Next, we apply a battery of unit root tests before applying the newly suggested test. Table 2 presents the traditional unit root test results.

Table 2. Results of traditional unit root tests

Countries	ADF unit root test		PP unit root test		KPSS stationarity test	
	Test Stat. (p-value)	Optimum Lag	Test Stat. (p-value)	Bandwidth	Test Stat.	Bandwidth
Brazil	−1.85 (0.356)	0	−1.852 (0.355)	3	0.297	15
China	−1.533 (0.516)	12	−1.263 (0.648)	5	1.459*	15
India	−1.293 (0.634)	15	−1.491 (0.537)	1	1.690*	15
Indonesia	−2.705 (0.074)***	14	−3.124 (0.026)**	9	0.295	15
Mexico	−1.934 (0.317)	14	−2.725 (0.071)***	2	0.834*	15
Russia	−1.763 (0.399)	11	−2.063 (0.26)	4	0.651**	15
Turkey	−0.279 (0.925)	11	−0.876 (0.795)	18	0.51**	15

Notes: *, **, and *** show significance at the 1%, 5%, and 10% levels, respectively. Numbers in the parentheses show the p-values.

Similar to the existing literature, the traditional unit root results provide little evidence of the validity of the PPP hypothesis. While the ADF test results show that the RER series are stationary for only Indonesia, the findings of the Phillips-Perron unit root test indicate that the RER series of Indonesia and Mexico are stationary, and finally, the results of KPSS stationarity test show that only RERs series of Brazil and Indonesia are stationary.

Next, to consider the nonlinearity in the data generation process of RERs, we apply the AESTAR unit root test of Sollis (2009) and the Fourier AESTAR unit root test that was introduced by Ranjbar *et al.* (2018) and tabulate the results in Table 3.

The AESTAR unit root test results, proposed by Sollis (2009), support the evidence of stationarity only RERs of Indonesia and Mexico. According to the Fourier AESTAR unit root test results, one frequency is optimal for all RER series, except for Brazil, which has two frequencies as optimal. Appendix supports the evidence that Fourier approximations fit well

Table 3. Results of nonlinear unit root tests

Countries	AESTAR		Fourier AESTAR			
	Test statistic	Optimum Lag	Optimum frequency	F-test statistic	Test statistic	Optimum Lag
Brazil	3.337	0	2	168.127	3.841***	0
China	1.651	12	1	1,219.930	8.150*	12
India	2.154	15	1	1,038.094	14.592*	15
Indonesia	32.734*	15	1	61.297	32.994*	15
Mexico	4.473***	9	1	372.153	3.300	9
Russia	1.431	6	1	228.532	6.428*	6
Turkey	2.036	0	1	527.009	3.347	1

Notes: * shows the significance at the 1 % level. The critical value for the F-test at the 1% level is 6.281. The critical values for the AESTAR unit root test at the 1, 5, and 10% levels are 6.236, 4.557, and 3.725, respectively. The critical values for the Fourier AESTAR unit root test for one frequency are 4.499, 3.894, and 3.556 and for two frequencies are 4.047, 3.515, and 3.264, at the 1, 5, and 10% levels, respectively.

with the large swings in the RERs. The results of the Fourier AESTAR unit root test show that the RER series of Brazil, China, India, Indonesia, and Russia are stationary. So, we can test the statistical significance of the Fourier function for the RERs of these countries. The F-test statistics in Table 2 support the evidence of smooth breaks in the data generation process of these RER series.

Finally, we examine the PPP hypothesis using the newly developed the FAESTAR-QKS unit root test, and Table 4 presents the results.

Table 4. Results of Fourier quantile AESTAR unit root test

Brazil				
Quantile	**Test statistics**	**10% CV**	**5% CV**	**1% CV**
0.1	41.557*	20.374	26.083	41.137
0.2	3.512	27.356	33.732	46.502
0.3	18.172	27.395	32.673	43.324
0.4	8.610	22.866	26.855	35.548
0.5	5.509	19.639	23.470	32.544
0.6	6.306	20.946	25.643	35.773
0.7	9.443	25.871	31.606	43.930
0.8	4.416	29.814	37.156	53.455
0.9	6.514	24.176	31.950	52.008
faestar-qks	53.603**	45.611	53.287	71.527
China				
Quantile	**Test statistics**	**10% CV**	**5% CV**	**1% CV**
0.1	7.910	20.112	26.061	41.137
0.2	4.712	27.247	33.340	45.801
0.3	8.523	26.158	30.873	40.284
0.4	10.152	22.302	26.104	34.630
0.5	10.629	20.543	24.478	33.696
0.6	15.924	21.684	26.553	37.110
0.7	26.953***	25.439	30.725	42.559
0.8	12.681	28.332	35.107	51.057
0.9	7.280	21.825	28.860	46.065
faestar-qks	36.844**	31.484	32.054	47.366

Table 4. Results of Fourier quantile AESTAR unit root test (continued)

India				
Quantile	Test statistics	10% CV	5% CV	1% CV
0.1	28.400***	22.715	30.195	50.330
0.2	8.060	27.398	34.136	47.798
0.3	10.923	22.584	27.888	39.328
0.4	89.390*	16.335	20.375	30.783
0.5	87.713*	15.118	18.940	27.650
0.6	15.740	18.741	22.250	31.711
0.7	43.876*	24.101	28.750	39.754
0.8	29.840***	26.611	32.482	46.771
0.9	4.824	21.576	28.088	42.771
faestar-qks	89.41522*	43.327	50.915	66.868
Indonesia				
Quantile	Test statistics	10% CV	5% CV	1% CV
0.1	9.003	36.382	49.251	82.574
0.2	91.627*	40.903	51.681	82.482
0.3	2.343	29.227	39.216	61.259
0.4	284.177*	14.946	22.363	46.822
0.5	255.486*	14.481	18.922	39.503
0.6	252.668*	20.284	26.910	59.969
0.7	328.416*	30.471	43.897	121.703
0.8	474.417*	41.267	66.071	163.072
0.9	5.785	42.942	65.347	135.397
faestar-qks	723.6715*	95.405	130.288	237.580
Mexico				
Quantile	Test statistics	10% CV	5% CV	1% CV
0.1	4.022	21.235	27.501	44.091
0.2	9.832	26.707	32.926	45.944
0.3	330.379*	23.018	28.065	39.196
0.4	266.536*	17.573	21.421	30.319
0.5	1.878	14.942	19.041	28.581
0.6	7.465	17.289	21.996	33.733

Table 4. Results of Fourier quantile AESTAR unit root test (concluded)

Mexico				
Quantile	Test statistics	10% CV	5% CV	1% CV
0.7	6.159	23.890	29.780	43.161
0.8	5.565	28.469	35.724	52.480
0.9	15.083	23.692	30.851	48.598
faestar-qks	361.784*	45.276	52.811	71.892
Russia				
Quantile	Test statistics	10% CV	5% CV	1% CV
0.1	21.427	23.232	30.169	48.184
0.2	53.492*	28.116	35.330	50.652
0.3	99.329*	24.061	29.426	42.622
0.4	7.410	16.598	21.013	31.101
0.5	1.567	13.564	17.697	27.067
0.6	0.780	17.123	21.448	31.989
0.7	3.510	23.567	29.100	41.138
0.8	7.338	27.793	34.095	49.022
0.9	8.459	22.027	28.227	45.013
faestar-qks	128.562*	45.373	52.994	69.824
Turkey				
Quantile	Test statistics	10% CV	5% CV	1% CV
0.1	19.383	21.451	27.671	42.732
0.2	26.845	27.589	33.676	47.018
0.3	5.104	24.574	29.682	41.266
0.4	28.085**	18.430	22.540	32.189
0.5	6.835	13.537	17.885	28.424
0.6	8.467	14.978	20.703	34.318
0.7	9.433	23.382	30.125	46.640
0.8	15.920	31.301	39.056	59.503
0.9	10.702	27.102	35.502	57.001
faestar-qks	30.5565	48.149	57.028	78.919

Notes: *, **, and *** show the significance at the 1, 5, and 10% levels. The critical values are obtained using 10,000 simulations.

Table 4 shows that the FAESTAR-QKS test provides more evidence for the PPP than the previous unit root tests. Accordingly, the FAESTAR-QKS unit root test supports the PPP hypothesis for all E7 countries except Turkey. However, the RER series of countries show different behaviors at different quantiles. Based on the results, Brazil shows the unit root behavior for nearly all quantile levels except for the lowest quantile, 0.1. For China, the null of the unit root cannot be rejected at all quantile levels except for the 0.7 quantile, which indicates the permanent impacts of positive and negative shocks. On the contrary, for Indonesia, we determine a stationary pattern for more than half of the quantiles. For India, unit root test results show a stationary pattern for the RER series for lower and middle quantile levels. Mexico and Russia exhibit similar unit root behavior. Finally, the results show that RER is stationary at 0.4 quantile level in Turkey. The quantile levels for the conditional distribution of economic variables indicate an economy's states, as Ma, Li and Park (2017) noted. For instance, low quantile levels indicate appreciation states and high quantile levels indicate depreciation states. Therefore, we can also interpret the results by considering the state of the economy. The RER series of Brazil, Mexico, and Russia show a stationary pattern for lower quantile levels, indicating that PPP holds at appreciation states. Besides, the PPP hypothesis mainly holds at medium and/or high quantile levels in China, India, and Indonesia; therefore, the PPP hypothesis holds at depreciation states for these countries.

5. CONCLUSION

Introducing a new quantile test, namely the Fourier quantile AESTAR unit root test, this paper tests the PPP theory for the emerging seven (E7) countries. We use monthly real exchange rate data from 1995:1 to 2023:6. First, we apply traditional and nonlinear unit root tests to test the PPP hypothesis. The results of these unit root tests show mixed results on the validity of the PPP hypothesis. Next, we test the PPP hypothesis using the Fourier quantile AESTAR unit root test. The results of the quantile unit root test show that the PPP hypothesis is valid for all countries in the sample except for Turkey. Besides, we observe that the RER of countries shows different unit root behavior at different quantile ranges.

The results reveal that PPP should be a crucial policy tool (or approach) for governments in six out of E7 countries, namely Brazil, China, India, Indonesia, Mexico, and Russia, to determine the equilibrium exchange rate. The policymakers of these countries can use PPP to predict the exchange rate that determines whether a currency is overvalued or undervalued; therefore, they can eliminate the differences between domestic and foreign inflation rates. In other words, economic policy advice derived from PPP models are appropriate for these countries.

More specifically, the calculation of the real values of the currencies in these countries has critical impacts on some economic policies, such as external competitiveness and/or external balance. According to the FAESTAR-QKS test results, the three countries in E7, namely Brazil, Mexico, and Russia, experience an appreciation in their currencies because the RER series show stationarity properties in the lower quantile levels. This means that currency appreciation can exacerbate current account imbalances in these countries, and therefore, they should avoid expansionary macroeconomic policies that could lead to inflation. On the other hand, China, India, and Indonesia experience depreciation in their currencies because the RER shows stationarity properties in the medium and/or high quantiles which means that currency depreciation can help these countries to reduce current account imbalances. Therefore, monetary and fiscal policies should not be inflationary; otherwise, depreciation will be eroded.

Finally, the Fourier AESTAR unit root test, which we have newly introduced to the literature, plays an important role in reaching these specific findings for emerging countries. Using this approach, future studies can investigate the validity of PPP for other countries (*i.e.* developed and frontier markets) and/or other currencies (*i.e.* euro, sterling) and therefore obtain more reliable results. ◄

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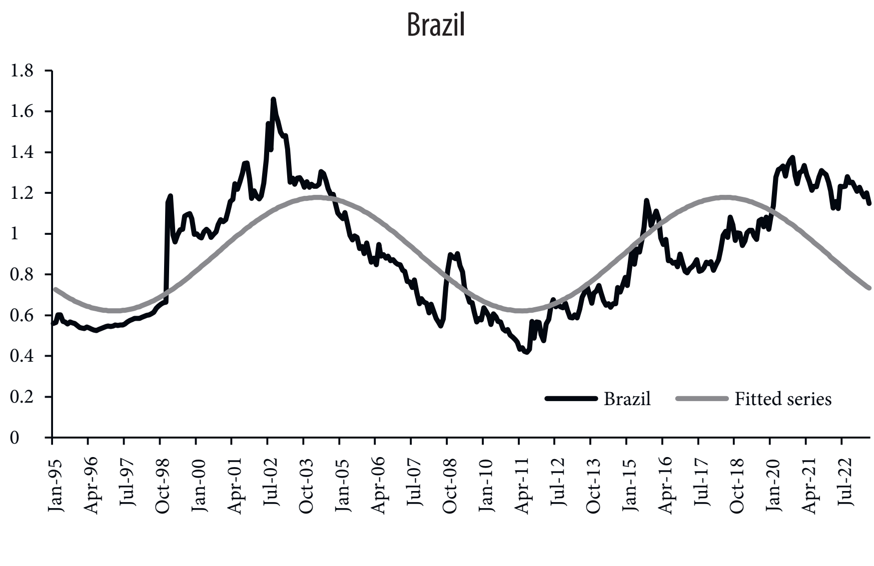
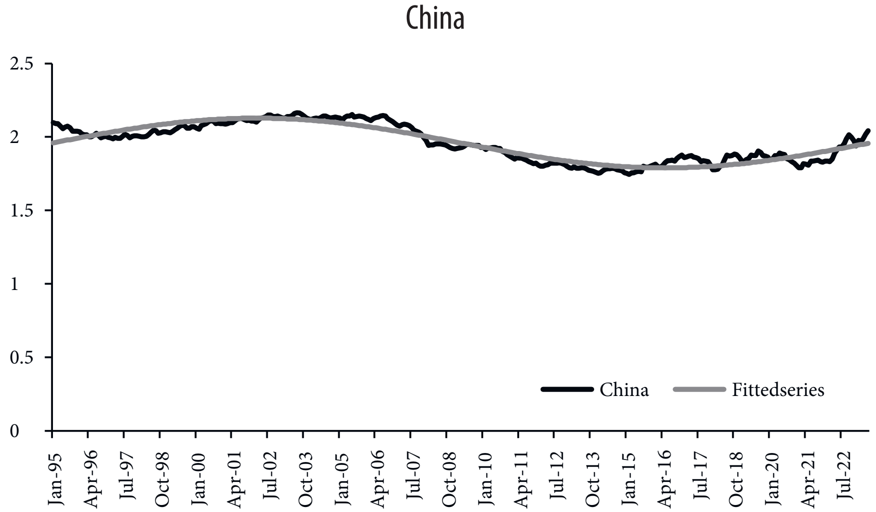
APPENDIX

RER series and estimated Fourier expansion

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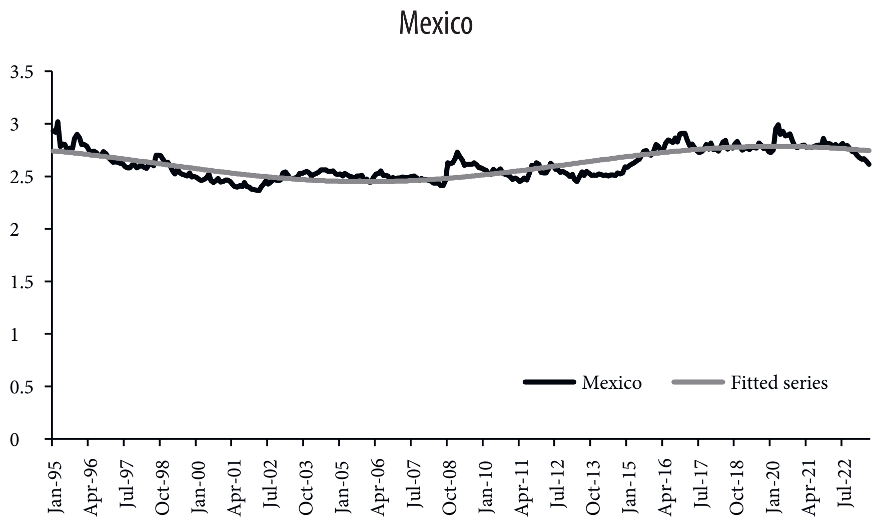
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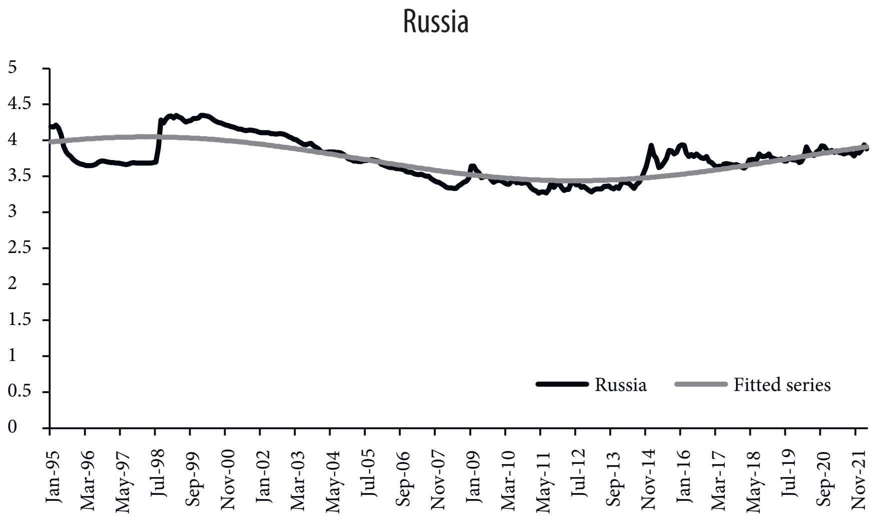
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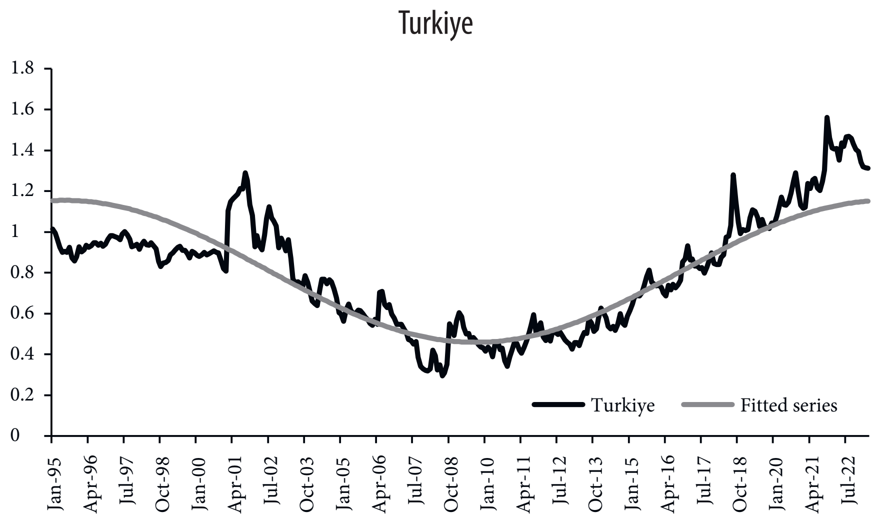
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