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Backtesting the Value at Risk for Latin American Stock and Currency Markets

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Abstract

In this article three methodologies are analyzed for calculating the value at risk (VaR): parametric, semi-parametric and non-parametric models. In order to evaluate their validity, a representative model was chosen for each: EGARCH for the parametrics, CAViaR for the semi-parametrics and the historic simulation for the non-parametrics. To validate these methodologies, the model proposed by Candelon *et al.* (2011) was used, a backtest based on the general method of moments. Variables to be forecast were the exchange rates and main stock-market indexes of the principal Latin American markets (Argentina, Brazil, Chile, Colombia, Peru, and Mexico). Results show that the CAViaR model is the best at forecasting the VaR for the markets and currencies during the periods that were analyzed.

Key words: value at risk,, backtest, parametric models, semi-parametric models, non-parametric models, GMM, CAViaR, EGARCH, HS.

JEL **Classification**: C14, C15, G10, G14.

Introduction

Decision-making under conditions of uncertainty is a pertinent and frequent concern in financial markets. Making decisions with greater information and knowledge is vital for many agents participating in these markets, especially when the main variable to be evaluated is not only risk but also returns.

For a long time risk has been quantified by a measure of dispersion (standard deviation or variance) that characterizes the volatility of an asset's return. A flaw in this method involves the inability to forecast future risks with some certainty. During the 1970s, some articles were published with analysis similar

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to those that in the 1980s would be formalized as value at risk (VaR). These studies sought to respond to the need to limit the uncertainty in forecasting an asset's risk and returns. Basically this methodology seeks to answer the following questions: How much loss can be expected in a day, a week, a month or a year, given a certain confidence or probability? What percentage of the investment's value is at risk?

This is the quantification, for a given confidence interval, of the amount or percentage of loss that an asset or a portfolio will face in a predefined time period (Jorion, 1997). Thus we can evaluate what an investment portfolio's minimum return will be next month with a 75% degree of confidence. So a critical value can be found, such that, according to the model's initial assumptions, a 75% probability exists that this return, or one greater, will occur. The application of the VaR is undertaken in investments, bank transactions, and project evaluations, among others. The analysis of the forecasted time period varies from a few minutes (in high-frequency operations), to years, depending on the application.

In 2004, the Basel II agreement granted financial institutions permission to create their own methods for administering risk. As a result, many of them have employed accepted academic techniques, such as Historical Simulation (HS), Conditional Autoregressive Value at Risk (CAViaR), and the Generalized Autoregressive Conditional Heteroscedasticity (GARCH) models, among others. In this context, it is important to determine which is the best model in accordance with its application and the period study. This article strives to address this matter.

To determine the most appropriate method, we undertook a compound backtest of three different tests: unconditional coverage (UC), independence (IND), and conditional coverage (CC), as stipulated under the methodology proposed by Candelon *et al.* (2011).

The analysis seeks to verify the validity of the distinct methodological families in assessing the VaR over different time periods. By so doing, we can observe changes in the soundness of those methodologies with respect to the economic situation of global markets, thus informing Latin American stock markets and decision-making agents therein.

After these introductory words, this paper consists of four sections: a review of relevant literature in which the fundamental concepts that underpin this study were developed; the data and methodologies used for analyzing Latin American markets; analysis of results; and deductions stemming from conclusions.

Review of literature

Investment risk analysis has been a fundamental concern since investing began. The first intuitive quantification of what is now known as VaR dates from Leavens (1945), who developed a quantitative example of the advantages of diversification. Later, Markowitz (1952) and Roy (1952) independently proposed measurements for the current VaR associated with portfolio selection, return optimization for a given level of risk, and estimations that incorporated covariances among the risk factors, in order to reflect the effects of coverage and diversification. Markowitz (1952) used a simple variation of returns, while Roy (1952) used a risk indicator that represented a higher limit on the probability of the portfolio's gross return. Later, several theoretical papers were written that suggested measurement of the VaR concept, without necessarily defining it: Tobin (1958), Treynor (1961), Sharpe (1964), Lintner (1965), and Mossin (1966). It was Dusak (1973) who described simple measurements of what today is known as VaR for futures portfolios, but without addressing the problems of stationality. Lietaer (1971) described a practical method for exchange rates. Garbade (1986) modeled risk measurements based on the sensitivity of bonds vis-à-vis returns, by assuming that market-portfolio values were normally distributed. Garbade (1987) took his own work further by introducing a way to reassign a diversified bond portfolio to a smaller one that consisted of only the more representative bonds, thus allowing the portfolio risk to be disaggregated.

Jorion (1997) formalized the concept of value at risk and defined it as the quantification of an amount or percentage of loss that a portfolio faces in a predefined period of time, given a certain level of significance or uncertainty; it should be noted, however, that formally it was Till Guldimann who created the concept during his tenure as head of international research for JP Morgan at the close of the 1980s.

According to Acerbi y Tasche (2002), the VaR does not satisfy the property of subadditivity of coherent risk measures for the diversification analysis of the sundry assets that make up an investment portfolio; likewise, Embrechts, McNeil, and Straumann (2002) showed that the VaR does indeed fulfill this property when the return on assets has a normal or T-student distribution. Although most distributions of asset returns do not fulfill this requirement, they can be transformed by a Cornish-Fisher expansion (Favre and Galeano, 2002), which, by means of kurtosis and skewness, creates a Z that very much resembles a normal Z.

Engle (1982) posited the autoregressive conditional heteroscedasticity (ARCH) models, beginning a new family of models that calculated a forecast's variability. Engle grounded his work in the heteroscedastic origin of the errors of the prediction models and suggested that these were autoregressive among themselves. With time, however, Bollerslev (1986) and Engle and Bollerslev (1986) extended the study by generalizing the ARCH models and indirectly established a method to calculate the VaR directly by means of the GARCH models.

In 1988, the Basel committee established the Basel I Accord that required banks to hold a minimum amount of capital equal to 8% of their total risk assets (the sum of credits, markets, and exchange rates). This was to be the first step in regulating bank risk. In 1989, JP Morgan, working through its research department, and in line with the Basel I Accord, created RiskMetrics®, that had its roots in Engle (1982) and Bollerslev's (1986) GARCH model, but with smoothed parameters that placed even greater importance on the most recent data than what had been the case.

Danielsson and Vries (2000) defined semi-parametric models in which the extreme value theory (EVT) and the CAViaR, set out by Engle and Manganelli (1999), appear. The advantage of CAViaR is that, when directly modeling the percentile of probability being studied, previous knowledge of the distribution of returns is not required. Table 1 shows complementary models that are used to calculate the VaR.

Allen and Singh (2010) applied the CAViaR to assess the risk of the Australian stock market, while Jeon and Taylor (2012) used it together with other models to forecast risk of the S&P500 and DAX30.

So y Yu (2006) empirically analyzed the ARCH models for VaR by means of the GARCH, IGARCH (integrated GARCH), FIGARCH (fractionally integrated GARCH), and RiskMetrics® for several indicators and Asian exchange rates. Angelidis, Benos and Degiannakis (2004) applied the GARCH, EGARCH (exponential GARCH), and TGARCH (threshold GARCH) models to the S&P500, Nikkei225, DAX30, CAC40, and FTSE100 indicators.

Christoffersen (1998) introduced the conditional-coverage hypothesis, which is divided into the unconditional-coverage hypothesis (the classical measure of the number of failures), as well as the independence hypothesis. The independence hypothesis, a value added, establishes that each "hit" (failure or violation) is independent of previous hits; this also involves analyzing the time between each hit (hitting time).

TABLE 1

Models used to calculate the VAR

Model	*Authors*	*Description*	*Advantages*	*Disadvantages*
Extreme value theory (EVT) models	Danielsson and de Vries (2000)	By means of non-parametric methods, this model studies the far ends of empirical distributions tails. The procedure involves smoothing the tail by means of a threshold index (M), allowing for an estimator to be calculated for the tail end, at a given confidence level, *i.e.*, a VAR.	Knowledge of empirical-data distribution is not required.	There are many differences in methods for estimating the threshold index (M), leading to several results depending on the option chosen.
Conditional autoregressive value at risk (CAVIAR)	Engle and Manganelli (1999)	Emphasizes modeling the probability percentile, combining parametric (autoregressive) with non-parametric (genetic algorithms) models.	Model is very adaptable to the type of data available.	Mixing parametric and non-parametric methods means calculations are not simple.
GARCH family models	Engle (1982) and Bollerslev (1986)	Autoregressively models the volatility of a sample, assuming the sample is not homoscedastic over time.	The level of parameters to be used can be estimated empirically, without requiring an assumption of homoscedasticity.	Assumes that the error distribution is normal or t-student, occasionally requiring expansions to use the data.
Exponentially weighted moving average (EWMA)	Special case laid out by JP Morgan (1989)	Models a sample's volatility based on its previous profitability and volatility, using factors (λ), which are imposed (as in the RiskMetrics case) or are calculated by a maximum likelihood estimation (general model).	Very simple calculations.	Assumes that the error distribution is normal or t-student, occasionally requiring expansions to use the data.
Historical simulation (HS)		Uses empirical data by ordering it from most to least, thus creating an empirical distribution. The VaR corresponds to the cumulated density of said distribution.	No elaborate methodology required for calculation.	When results are analyzed over time, it appears to not be very sensitive to changes in data.

Table 1, continuation…

Model	*Authors*	*Description*	*Advantages*	*Disadvantages*
Monte Carlo Simulation(MC)	Metropoli and Ulam (1949) Forerunners: Fermi, Ulam, Von Neumann, Metropoli.	Simulates N scenarios where data can change, thus generating a simulated distribution. The VAR will correspond to α% of that distribution.	Takes into account a large number of positive and negative future scenarios.	The stochastic process is difficult to characterize correctly due to many statistical conditions that need to be accounted for.
Higher-order Conditional Time-varying moments	Bali, Mo, and Tang (2008)	This methodology relaxes the assumption that the distribution of returns is identically and independently distributed (iid), since it is stipulated that moments such as kurtosis, skewness, tail thickness, etc., are variable over time and not constant, which in the end yields a better calibration of data for calculating the VAR.	Relaxes the assumption that the distribution of returns is iid.	Better data is needed than other models, which may imply difficulties when done on a monthly or annual basis.
Volatility-Weighted Historical Simulation	Hull and White (1998)	Updates the information on returns obtained (after ordering them as in the basic historical simulation) and adjusts the returns by future forecasted volatility. In the end, up-to-date data are obtained by means of their volatility, and the VaR is calculated analogously to the previous HS.	Takes into account the changes in the volatility of returns, undertaking more exact calculations than the previous HS.	Depends on how volatility is forecast, which can vary depending on how the user forecasts this volatility.

Christoffersen y Pelletier (2004) prepared a test that combines the two hypotheses (to test the two necessary properties to validate a projection) that were posited by Christoffersen (1998) in just one overarching hypothesis. Berkowit, Christoffersen, and Pelletier (2009) ground their article in Christoffersen and Pelletier (2004) to further study their own test, which is based on the time between hits. Candelon *et al.* (2011) employ what Berkowitz proposed, but are able to separate the hypotheses that Christoffersen and Pelletier had unified, setting out the two initial hypotheses and the joint hypothesis in one. Candelon *et al.* (2011) subject their test to a check for robustness, flexibility, and exactness of several properties (types of samples, size, etc.), achieving positive results, which, at that time, placed this check at the leading edge of VaR tests.

Data and methodology

The securities to be analyzed are the principal Latin American currencies: the Brazilian real, the Argentine, Chilean, Colombian, and Mexican pesos, and the new Peruvian sol, all expressed in local currency per US dollar. The stock indexes analyzed are: the Securities Market of Buenos Aires (*Mercado de Valores de Buenos Aires, S.A.*, Merval) of Argentina; the São Paulo Securities Exchange (*Bolsa de Valores de São Paulo*, Bovespa) of Brazil; the Index of Selective Price of Stocks (*Índice de Precio Selectivo de Acciones*, IPSA) of Chile; the General Index of the Securities Exchange of Colombia (*Índice General de la Bolsa de Valores de Colombia*, IGBC), the Index of Prices and Values (*Índice de Precios y Cotizaciones*, IPyC) of Mexico; the General Index of the Securities Exchange of Lima (*Índice General de la Bolsa de Valores de Lima* (IGBVL) of Peru; the Dow Jones Industrial Average (DJIA) and the index of National Association of Securities Dealers Automated Quotation (Nasdaq) of United States. Data from January 2, 1990 to May 31, 2012 were obtained from the *Economática* data base. The IGBC index in Colombian currency is available from January 2, 1991 to January 4, 1993 (in dollars). The Argentine peso was pegged until 2002, and thus the corresponding currency index was evaluated with post-2002 data.

Analyses were undertaken for four time periods, each divided into two sub-periods: one for model building, the other for forecasting, taking into consideration that the time relationship between these two sub-periods is 2:1. The first period (period 1) runs from January 1990 to December 1996 (Jan-90 and Dec-96, respectively), which analyzes the model's behavior in the pre-crisis period; the second period (period 2), from January 1991 to December 1999 (Jan-91 and Dec-99) illustrates the models forecasted during the Asian crisis; the third period (period 3) runs from January 2000 until May 2012 (Jan-00 and May-12) and aims to forecast during the subprime crisis with formation data compiled previous to the crisis. Finally, the fourth period (period 4), runs from January 2007 to May 2012 (Jan-07 and May-12) and aims to test the models in times of high volatility, both during the model-building period as well as in forecasting. Table A1 shows the periods under analysis.

The methodologies to be applied in the study are HS, CAViaR, and GARCH. Each one represents a type of VaR calculation: non-parametric, semi-parametric, and parametric. The historical simulation will use the returns from the index observed in one time period in order to determine the series of changes in its value, considering that the VaR of that period is equal to the percentile of the distribution of returns given the desired confidence level. This will be done through moving windows of 250 days, which are approximately equivalent to one year.

For the direct application of the GARCH model to the VaR calculation, although Engle and Manganelli (2001) posit that a GARCH(1,1) is more appropriate, we chose to use an EGARCH, as proposed by Nelson (1991), since this allows us to incorporate the skewness effect (see equation [1]). For each formation period, we optimized the best model, comparing the Akaike Information Criterion (AIC) for all possible models with different s, p, and q, and each lag had a maximum value of 5.

$$r_t = \sum_{i=1}^{s} \alpha_i r_{t-i} + \varepsilon_t \qquad [1]$$

$$\ln(\sigma_t^2) = \gamma + \sum_{j=1}^{p} \beta_j \left| \frac{\varepsilon_{t-j}}{\sigma_{t-j}} \right| + \sum_{k=1}^{p} \delta_k \frac{\varepsilon_{t-k}}{\sigma_{t-k}} + \sum_{l=1}^{q} \varphi_l \ln(\sigma_{t-l}^2)$$

The CAViaR model, set out by Engle and Manganelli (1999), focuses on modeling the probability percentile. To do so, we first consider that the return on securities tends to merge with time, *i.e.*, correlation among them will occur. The parameters of this model are estimated by means of regressions by quantiles, following Koenker and Bassett (1978). The general definition of the CAViaR is:

$$VaR_t = f(x_t, \beta_\theta) = \beta_0 + \sum_{i=1}^{p} \beta_1 VaR_{t-1} + l(\beta_{p+1}, \ldots, \beta_{p+q}; \Omega_{t-1}) \quad [2]$$

where Ω_{t-1} is the set of information available at moment t.

Note that in most practical cases, this expression can be linearized and expressed in a simpler form:

$$VaR_t = \beta_0 + \beta_1 VaR_{t-1} + l(\beta_2, y_{t-1}, VaR_{t-1}) \quad [3]$$

The autoregressive term $\beta_1 VaR_{t-1}$ guarantees that the changes in the VaR are smooth over time; likewise, $l(\beta_2, y_{t-1}, VaR_{t-1})$ shows the relationship between the level of VaR_t and that of y_{t-1}, *i.e.*, it measures how much the VaR should change as a function of new information at y. Note that this term plays a role identical to the impact curve from the GARCH models introduced by Engle and Ng (1993).

The backtest is divided into three different tests: UC, IND, and CC. The CC test encompasses the other two, but the advantage of running the three is that if the CC is rejected, we can see if it was rejected due to UC or IND. These tests require that the null hypothesis not be rejected, and therefore require a p[1] value greater than the designated security value. Another parameter to be considered is the number of polynomials used in the (P) test. This number is important in determining if the hypotheses are rejected or not, given that the higher it is, the higher the degree of polynomials in the statistic, making it more precise and giving it greater accuracy. In this study the p value to be used is 10% and the number of polynomials is 6, which are values used by Candelon *et al.* (2011). All these tests are based on "hits", a binary variable that is activated when the VaR has been violated:

$$I_t(\alpha) = \begin{cases} 1 \text{ if } r_t < VaR_{t|t-1}(\alpha) \\ 0 \text{ otherwise} \end{cases}$$

[1] This is the probability of obtaining at least a similar result to that obtained by calculating the statistic (Greene, 2002).

Regarding this variable, Christoffersen (1998) determined that the VaR predictions are valid if and only if the hit sequence [$I_t(\alpha)$] satisfies the unconditional-coverage property and the independence property. The unconditional-coverage property refers to the probability that an *a-posteriori* return would be greater than forecasted; the VaR should be equal to the hedge rate of α%:

$$P[I_t(\alpha) = 1] = E[I_t(\alpha)] = \alpha \qquad [5]$$

The property of independence is associated with violations of the VaR. The variable $I_t(\alpha)$ associated with a violation of the VaR at time t with a α% hedge rate must be independent of the variable $I_{t+k}(\alpha)$ for all k not equal to zero.

When the UC e IND properties are simultaneously valid, we say that the VaR forecasts have correct conditional coverage, and the process of VaR violation is a martingale process. The statistics for this hypothesis are based on d_i, *i.e.*, the time between two consecutive violations:

$$d_i = t_i - t_{i-1} \qquad [6]$$

where t_i is the date of the *i*-th violation.

Note that the backtest meets the conditions of the moments based on orthonormal polynomials, and thus is defined as a sequence of time durations between N violations of the VaR, $\{d_1, d_2, \ldots, d_N\}$, which are calculated by means of a sequence of $I_t(\alpha)$ hit variables. Under the assumption of conditional coverage, the duration d_i, $i = 1,\ldots,N$, is independently and identically distributed (iid), and has a geometric distribution with a probability of success equal to the hedge rate α. Thus the CC null hypothesis can be expressed by equation [7].

$$H_{0,CC}: E[M_j(d_i;\beta)] = 0, j = 1,\ldots,p \qquad [7]$$

where p expresses the number of conditional moments. Thus the UC null hypothesis can be expressed by equation [8], and the IND null hypothesis by equation [9]:

$$H_{0,UC}: E[M_1(d_i;\alpha)] = 0 \qquad [8]$$

$$H_{0,IND}: E[M_j(d_i;\beta)] = 0, j = 1,\ldots,p \qquad [9]$$

Equation [9] shows that the duration between two consecutive violations has a geometric distribution. Note that the UC hypothesis does not hold if β is not equal to α.

Now, following Bontemps and Meddahi (2006), the orthonormal polynomials have an advantage insofar as their asymptotic covariance matrix is known. Under the criteria of the Generalized Method of Moments (GMM), the optimal-weight matrix is simply an identity matrix in which $J_{CC}(p)$ denotes the CC statistical test associated with the first p orthonormal polynomials. Assuming that the duration $(d_i = 1 < i)$ is stationary and ergodic, the conditional coverage $J_{CC}(p)$ null hypothesis would be expressed thusly:

$$J_{CC}(p) = \left(\frac{1}{\sqrt{N}} \sum_{i=1}^{N} M(d_i;\alpha) \right)^T \left(\frac{1}{\sqrt{N}} \sum_{i=1}^{N} M(d_i;\alpha) \right) \rightarrow^{d}_{N\rightarrow\infty} X^2(p) \qquad [10]$$

where $M(d_i;\alpha)$ represents a $(p,1)$ vector, whose components are the ortonormal polynomials $M_j(d_i;\alpha)$ for $j = 1,\ldots,p$, and α indicates the α% rate of coverage.

The UC test statistic, J_{UC}, is obtained as a special case of the J_{CC} statistic, when just the first orthonormal polynomial is considered, in other words, when $M(d_i;\alpha) = M_1(d_i;\alpha)$. Then J_{UC} is equal to $J_{CC}(1)$ and we obtain the following expression:

$$J_{UC}(p) = \left(\frac{1}{\sqrt{N}} \sum_{i=1}^{N} M_1(d_i;\alpha) \right)^2 \rightarrow^{d}_{N\rightarrow\infty} X^2(1) \qquad [11]$$

Lastly, the IND statistic, J_{IND}, can be expressed by equation [12]:

$$J_{INP}(p) = \left(\frac{1}{\sqrt{N}} \sum_{i=1}^{N} M(d_i;\beta) \right)^T \left(\frac{1}{\sqrt{N}} \sum_{i=1}^{N} M(d_i;\beta) \right) \rightarrow^{d}_{N\rightarrow\infty} X^2(p) \qquad [12]$$

where $M(d_i;\beta)$ denotes a $(p,1)$ vector, whose components are the orthonormal polynomials $M_j(d_i;\beta)$ for $j = 1,\ldots,p$ as evaluated for a probability of success equal to β.

Note that all p values produced by this test are corrected by means of Dufour's correction optimizing process (Dufour, 2006).

Analysis of results

The best analysis of results should account for the descriptive statistic associated with the forecast periods, for both stock indexes in national currency and in dollars (see Table A2 in the appendix), as well as for exchange rates (see Table A3 in the appendix). For the first forecast period (period 1) we can see that the average return is not homogeneously positive. The standard deviation is less here than in the Asian crisis (period 2), explained by the fact that period 1 is pre-crisis. We can also see a positive skewness in almost all indexes, except the IPyC in dollars, the Nasdaq, and the DJIA.

The second period, corresponding to the Asian crisis, is characterized by greater volatility. Almost all series have a negative skewness, except the IGBC and the IPSA, both in dollars. There is also leptokurtic behavior in all series, which is repeated in all periods. The Mexican peso is less volatile than in the pre-crisis period. Again, in general the exchange rates have an opposite sign in symmetry when compared with their stock indexes. In the third period, corresponding to the subprime crisis, we see a similar behavior in the stock indexes as in the Asian crisis, with similar standard deviation values. With regard to the exchange rates, they show greater volatility than in the previous crisis period, with the exception of the new Peruvian sol (PEN). We can see that those exchange rates with high volatility have differences in their indexes in national currency and in dollars, specifically the Mexican peso (MXN) and the IPyC.

Finally, in the fourth period, associated with the forecast in a volatile stage with a volatile formation, the standard deviation of all stock-return series is less than the complete crisis period (period 3), associated with the recovery that took place towards the end of the period. We find negative asymmetric and leptokurtic behavior in all series. We see the same behavior in the exchange rates, less volatility, while skewness is positive in all series, except for the Colombian peso (COP). All behave in a leptokurtic manner.

We can see in Table 2 the results of the backtest applied to the forecast made for the various GARCH, HS y CAViaR models, for the periods under analysis. In period 1, five of the stock markets have valid models, with CAViaR prevailing as the model with the best fit. During this period the exchange rates are not very volatile (except MXN), for which reason we should not be surprised that in valid models the Merval and the IPSA are equal, both in national currency and in dollars. In the case of the IGBC, we should recall that the projected data

Table 2
Valid models for stock indexes in national currencies and international currencies (dollars) and rates of exchange by period

	Period 1	*Period 2*	*Period 3*	*Period 4*
Stock indexes				
National currency				
Bovespa		CAViaR		
IGBC				CAViaR/HS/EGARCH
IGBVL				EGARCH
IPSA	CAViaR	CAViaR		CAViaR
IPyC				HS/EGARCH
Merval	CAViaR	CAViaR	EGARCH	EGARCH
International currency (dollar)				
Bovespa				
IGBC	CAViaR			EGARCH
IGBVL				
IPSA	CAViaR	CAViaR		EGARCH
IPyC				EGARCH
Merval	CAViaR/EGARCH	CAViaR		CAViaR/EGARCH
DJIA	CAViaR/HS			
Nasdaq	CAViaR			
Rates of exchange				
Brazilian real			CAViaR/EGARCH	
Colombian peso	CAViaR			EGARCH
New Peruvian sol			CAViaR	EGARCH
Chilean peso	CAViaR/EGARCH			CAViaR
Mexican peso				CAViaR
Argentine peso	*	*		

Note: */ not applicable since a fixed exchange rate prevailed during part of the period: 1 Argentine peso for 1 U.S. dollar. This table shows only valid models for each of the indexes and exchange rates in the six periods studied. To be valid they must pass the UC, IND y CC tests.

are not equal in number, given than they are available in dollars since 1993, and, unsurprisingly, the results differ even with an exchange rate with a small standard deviation. Note that the IPyC and Bovespa have the largest standard deviations in the period, which may indicate that there is an upper limit to this characteristic of the series in order for the VaR forecasts to be valid. In terms of the exchange rates, we note that the Chilean peso (CLP) and Colombian peso (COP) have valid models, while the Argentine peso (ARS) was not analyzed due to the pegged exchange rate at that time. Interestingly, during this period two exchange rates that do not have valid models are the most and least volatile therein, leading us to conclude that there is an upper and lower limit to the volatility of variables in order to have valid models in the forecast of their corresponding VaR. Regarding the PEN we see that the IND and UC hypotheses in the GARCH were barely rejected, which makes the PEN dependent on the degree of confidence required in the statistical test.

In the second period, associated with the Asian crisis, there is generally greater volatility, leading to differences in results for Bovespa's national currency and dollars. The currency's descriptive statistic is among the most volatile during the period, which might explain the difference in the results of the indexes therein; as in the previous period, the IPSA and Merval maintain CAViaR as a valid model, both for the national currency and the dollar. In this case, the descriptive statistic shows differences between the DJIA and Nasdaq, which is logical given the crisis during the period, which had a diverse impact on quite a few sectors of the U.S. market. In fact, it was during this period that the dot-com bubble burst. Neither the DJIA nor Nasdaq has a valid model during this period. By analyzing the tests (see Table A4 in the appendix), we can see that the vast majority of them reject the IND but not the UC as in previous periods. This indicates that the hits (or violations) are not independently distributed, which can be confirmed given the period at hand. Note that the IGBC and IGBVL indexes show zero violations in some models, thus confirming the smooth tendency of their returns.

Further, no currency obtained valid models in their projection of the VaR, which can be explained by their descriptive statistic, given that, although a greater variability exists, there is greater skewness among the high-variability currencies. This is apparently why the BRL was unable to obtain valid models in spite of its elevated dispersion, given that its skewness coefficient is very high when compared to the mean among currencies.

The period associated with the subprime crisis is much longer than the Asian crisis. We can see that the former has fewer valid models for the indicators. Among currencies, only the Brazilian real and the new Peruvian sol have valid models, both of them CAViaR; and in particular, the real also has the EGARCH.

The last period to be analyzed encompasses the subprime crisis, and thus in formation with crisis, in which the HS model figures as valid in two markets, indicating that by having a more volatile data projector, the probably of it being valid increases. The IGBC has two valid models, showing that the high standard deviation of the forecasting data produces a better VaR projection.

We can observe a difference in the validity of the models in terms of the indicators in national currency and in dollars. This might be an effect of the volatility present in the forecasting data of the exchange rates, which could lead to divergences in both indexes. When we examine the tests of indicators in dollars, they fail slightly more in the UC hypothesis, thus confirming the lack of, or excess of, violations in the VaR.

Conclusions

After analyzing results from forecasts made with the various methodologies and the backtest, we conclude that the CAViaR is the most accurate method to use with exchange rates in all periods. For stock market forecasts, the CAViaR, together with the EGARCH model, produce the largest amount of correct projections. This demonstrates that the parametric and semi-parametric models show similarities in their calculations, and both have an autoregressive component.

The historical simulation turned in a very unsatisfactory performance for the markets and the periods analyzed herein. It was only accurate in the final period for two stock indexes in which a greater variability exists in the real data that cover the forecasted period. The Peruvian stock index and the Mexican peso could only be modeled in the final period.

There is a prevalence of the CAViaR modes in forecasts of stock indexes before the Asian crisis, and a homogeneity of GARCH models before the subprime crisis. During this latter period, only the CAViaR model seemed valid for exchange rates.

In the pre-crisis period, most methodologies rejected only the UC test, indicating an excess or lack of VaR violations corresponding to the required 5% confidence level, which points to the presence of limits on the standard deviation of data to be forecast. In terms of the IND test, it was largely rejected in the crisis periods, as explained by the changing economic variables inherent to any crisis period, and which are omitted by these methodologies.

We observed the presence of leptokurtosis in most of the series considered, which supports the use of a GARCH model with a t-student distribution in its errors.

Given these results, we note the importance of historical data for calculating future risk, in both crisis and normal periods. This fact can help in estimating volatility when an economic policy decision is being made, such as intervening in the exchange rate or modifying monetary policy (stimulating or discouraging savings). This, plus the approved methodologies, can produce a more precise estimation of what will occur in the future and decrease the risk that funds could encounter. Better tuned regulations could strengthen the banking, finance, and insurance sectors, as well as the pension-fund administration sector, among others.

The final conclusion is that, in general, the CAViaR methodology is the better of the three models for calculating the VaR of stock and Latin American exchange-rate indexes for the periods under study. Currently, numerous regulations designed to control fund risk require calculating the VaR as a control metric, but not all regulations define a methodology for calculating it. Therefore, the CAViaR model should be proposed as a methodology, since it is essential when regulating risk of funds administered by banks or financial institutions.

In summary, we believe it is important that legislators define risk when faced with laws that seek to limit it in investments or in regulating bank operations. Further, from the point of view of agents in the Latin American stock or exchange-rate markets, this is a relevant factor in making risk-return decisions, in deciding the extent of risk exposure and in finding ways of lessening it. The quantification of risk is a necessary step in making decisions in an atmosphere of uncertainty, and this paper helps increase knowledge in the region in this regard.

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Appendix

Table **A1**
Formation and forecast periods for time periods analyzed

Period	*Formation*			*Forecast*			*Objective*
Period 1	Jan-90	:	Dec-94	Jan-95	:	Dec-96	Pre-crisis period
Period 2	Jan-91	:	Dec-96	Jan-97	:	Dec-99	Asian crisis forecast
Period 3	Jan-00	:	Dec-07	Jan-08	:	May-12	Subprime crisis forecast
Period 4	Jan-07	:	May-10	Jun-10	:	May-12	Formation and forecast in crisis

Note: this table lists the periods under analysis in this paper, differentiating the formation and forecast periods for each time period. Also included is the objective or characterization for conclusion purposes.

TABLE **A2**

Descriptive statistics of stock indexes for forecast periods

	Bovespa		IGBC		IGBVL		IPSA		IPYC		Merval		Nasdaq	DJIA
	BRL	USA	COP	USA	PEN	EE.UU.	CLP	USA	MXN	USA	ARS	USA	USA	USA
Period 1														
Average (%)	0.091	0.049	–0.011	–0.050	0.004	–0.031	–0.019	–0.028	0.072	–0.009	0.067	0.070	0.113	0.109
Standard deviation (%)	2.757	2.817	0.932	0.987	1.593	1.673	1.278	1.362	1.866	2.553	2.398	2.407	0.908	0.649
Skewness	1.09	1.09	0.41	0.00	0.25	0.23	0.84	0.75	0.29	–0.83	0.05	0.07	–0.70	–0.58
Kurtosis	14.57	15.54	5.03	5.24	9.69	10.02	10.28	9.40	6.55	10.23	6.01	6.07	5.18	5.32
Period 2														
Average (%)	0.098	0.018	0.023	–0.063	0.032	–0.010	0.019	–0.015	0.084	0.061	–0.022	–0.022	0.126	0.071
Standard deviation (%)	3.214	3.275	1.382	1.520	1.297	1.369	1.487	1.565	2.006	2.104	2.432	2.436	1.549	1.167
Skewness	0.62	–0.19	0.48	0.18	–0.26	–0.41	0.29	0.17	–0.03	–0.39	–0.77	–0.77	–0.52	–0.54
Kurtosis	15.26	8.18	10.67	9.95	6.91	7.58	7.62	7.70	10.83	9.76	9.26	9.30	5.84	7.33
Period 3														
Average (%)	–0.006	–0.015	0.029	0.037	0.025	0.035	0.026	0.009	0.007	–0.026	0.007	–0.026	0.013	–0.002
Standard deviation (%)	2.112	2.795	1.314	1.560	2.040	2.174	1.579	2.126	2.176	2.205	2.176	2.205	1.818	1.597
Skewness	0.05	–0.24	–0.51	–0.62	–0.46	–0.39	0.22	0.01	–0.60	–0.61	–0.60	–0.61	–0.19	–0.01
Kurtosis	9.41	11.32	9.92	6.91	10.26	10.41	8.81	9.67	7.76	7.98	7.76	7.98	7.67	9.50
Period 4														
Average (%)	–0.032	–0.051	0.034	0.049	0.081	0.096	0.030	0.047	0.041	0.035	0.005	–0.022	0.055	0.051
Standard deviation (%)	1.395	1.809	1.097	1.220	1.677	1.754	1.103	1.343	1.053	1.500	1.761	1.758	1.369	1.126
Skewness	–0.51	–0.64	–0.26	–0.54	–1.13	–1.08	–0.59	–1.03	–0.59	–0.75	–0.70	–0.70	–0.44	–0.48
Kurtosis	6.18	5.41	4.31	4.06	13.84	13.47	10.50	9.40	7.04	7.02	7.48	7.47	5.96	6.35

Note: USA: US dollar; BRA: Brazilian real; COP: Colombian peso; PER: new Peruvian sol; CLP: Chilean peso; MXN: Mexican peso; ARS: Argentine peso. Statistical indicators are shown that characterize various stock-index series, grouped by periods of analysis. Descriptive statistics are calculated for stock indexes in both national currency and in international currency (US dollar).

Table A3
Descriptive statists for exchange rates during forecast periods

	ARS	BRL	CLP	COP	MXN	PEN
Period 1						
Average (%)		0.0413	0.0101	0.0393	0.0784	0.0346
Standard deviation (%)		0.2600	0.3685	0.3010	1.9483	0.2790
Skewness		1.60	0.25	0.39	1.39	–0.04
Kurtosis		16.76	6.33	7.08	21.30	6.19
Period 2						
Average (%)		0.0733	0.0289	0.0857	0.0255	0.0406
Standard deviation (%)		0.9898	0.3572	0.5588	0.6795	0.3656
Skewness		2.95	0.64	1.13	0.69	–0.19
Kurtosis		50.55	12.50	15.49	18.78	52.46
Period 3						
Average (%)	0.0322	0.0121	0.0039	–0.0085	0.0242	–0.0092
Standard deviation (%)	0.1835	1.1238	0.8166	0.8873	0.9114	0.3537
Skewness	1.94	0.38	0.66	–0.12	0.87	0.31
Kurtosis	38.05	15.20	7.36	8.54	16.98	15.34
Period 4						
Average (%)	0.0267	0.0215	–0.0043	–0.0154	0.0202	–0.0097
Standard deviation (%)	0.0698	0.7596	0.6657	0.5460	0.8028	0.1747
Skewness	0.81	0.60	1.23	–0.26	1.05	0.11
Kurtosis	6.60	6.63	9.82	9.89	23.93	15.14

Note: statistical indicators are shown that characterize the exchange-rate series, grouped by period of analysis.

Table **A4**
Minimum p-value for each test

Minimum *p*-value for stock-index tests in national currency

Stock indexes	*Period 1*		*Period 2*		*Period 3*		*Period 4*	
Bovespa								
CAViaR	0.000	0.000	0.162	0.102	0.004	0.000	0.000	0.000
EGARCH	0.115	0.047	0.022	0.220	0.005	0.000	0.006	0.003
HS	0.001	0.002	0.000	0.000	0.012	0.004	0.203	0.078
IGBC								
CAViaR	0.007	0.005	0.008	0.000	0.002	0.645	0.458	0.218
EGARCH	0.006	0.006	0.001	0.000	0.011	0.006	0.805	0.464
HS	0.019	0.009	0.005	0.000	0.002	0.000	0.305	0.288
IGBVL								
CAViaR	0.005	0.001	0.025	0.007	0.015	0.013	0.032	0.075
EGARCH	0.002	0.030	0.005	0.004	0.006	0.001	0.206	0.594
HS	0.002	0.001	0.000	0.000	0.006	0.000	0.004	0.000
IPSA								
CAViaR	0.919	0.408	0.192	0.708	0.000	0.000	0.551	0.302
EGARCH	0.229	0.090	0.013	0.006	0.001	0.008	0.146	0.023
HS	0.001	0.001	0.003	0.000	0.018	0.011	0.012	0.000
IPyC								
CAViaR	0.006	0.002	0.001	0.002	0.073	0.019	0.000	0.022
EGARCH	0.215	0.088	0.066	0.044	0.030	0.574	0.208	0.190
HS	0.003	0.001	0.016	0.002	0.000	0.000	0.195	0.107
Merval								
CAViaR	0.569	0.598	0.497	0.336	0.015	0.006	0.108	0.041
EGARCH	0.197	0.083	0.044	0.045	0.199	0.636	0.528	0.314
HS	0.001	0.000	0.006	0.001	0.019	0.008	0.072	0.017

Note: a minimum *p*-value is shown for each stock index and its national currency analysis for each UC and IND test. If the minimum *p*-value is greater than 10%, the null hypothesis cannot be rejected. If both tests are not rejected, the model is considered valid for the relevant index and the particular period under study.

Table **A4, continuation…**

Minimum *p*-value for stock-index tests in US (international) currency

Stock indexes	*Period 1*		*Period 2*		*Period 3*		*Period 4*	
Bovespa								
CAViaR	0.000	0.000	0.086	0.031	0.001	0.000	0.000	0.002
EGARCH	0.000	0.000	0.017	0.006	0.034	0.028	0.165	0.080
HS	0.001	0.003	0.003	0.000	0.001	0.000	0.023	0.005
IGBC								
CAViaR	0.104	0.205	0.023	0.024	0.034	0.085	0.002	0.002
EGARCH	0.206	0.092	0.007	0.014	0.008	0.038	0.231	0.362
HS	0.025	0.014	0.009	0.002	0.000	0.000	N/A	N/A
IGBVL								
CAViaR	0.000	0.003	0.004	0.005	0.003	0.599	0.007	0.015
EGARCH	0.000	0.001	0.000	0.001	0.012	0.003	0.030	0.185
HS	0.000	0.002	0.000	0.000	0.000	0.000	0.003	0.002
IPSA								
CAViaR	0.556	0.236	0.291	0.220	0.001	0.004	0.035	0.046
EGARCH	0.016	0.238	0.001	0.016	0.000	0.010	0.882	0.427
HS	0.001	0.001	0.009	0.003	0.004	0.000	0.017	0.001
IPyC								
CAViaR	0.037	0.295	0.087	0.036	0.114	0.078	0.034	0.026
EGARCH	0.057	0.103	0.091	0.037	0.008	0.277	0.284	0.171
HS	0.000	0.002	0.019	0.004	0.021	0.004	0.028	0.009
Merval								
CAViaR	0.752	0.541	0.585	0.371	0.022	0.012	0.164	0.140
EGARCH	0.193	0.220	0.033	0.035	0.014	0.082	0.157	0.139
HS	0.001	0.000	0.006	0.001	0.009	0.003	0.110	0.044
DJIA								
CAViaR	0.571	0.505	0.000	0.035	0.000	0.000	0.248	0.075
EGARCH	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001
HS	0.286	0.119	0.056	0.020	0.016	0.011	0.006	0.001
Nasdaq								
CAViaR	0.183	0.236	0.000	0.094	0.022	0.036	0.084	0.047
EGARCH	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001
HS	0.106	0.018	0.061	0.029	0.000	0.000	0.002	0.001

Note: a minimum *p*-value is shown for each stock index and its national currency analysis for each UC and IND test. If the minimum *p*-value is greater than 10%, the null hypothesis cannot be rejected. If both tests are not rejected, the model isconsidered valid for the relevant index and the particular period under study.

Table A4, continuation…

Minimum p-value for Latin American exchange-rate tests

Exchange rate	Period 1		Period 2		Period 3		Period 4	
	UC	IND	UC	IND	UC	IND	UC	IND
Brazilian real								
CAViaR	0.000	0.000	0.064	0.026	0.845	0.421	0.000	0.000
EGARCH	0.000	0.000	0.000	0.000	0.924	0.406	0.000	0.000
HS	0.000	0.000	0.007	0.001	0.000	0.000	0.001	0.000
Colombian peso								
CAViaR	0.294	0.171	0.000	0.223	0.145	0.092	0.000	0.000
EGARCH	0.543	0.010	0.086	0.271	0.021	0.011	0.278	0.101
HS	0.025	0.002	0.018	0.005	0.015	0.008	0.038	0.020
New Peruvian sol								
CAViaR	0.001	0.001	0.000	0.000	0.381	0.203	0.001	0.002
EGARCH	0.002	0.265	0.000	0.000	0.019	0.008	0.517	0.186
HS	0.000	0.001	0.004	0.002	0.000	0.000	0.031	0.007
Chilean peso								
CAViaR	0.605	0.399	0.017	0.006	0.006	0.134	0.187	0.104
EGARCH	0.193	0.116	0.001	0.008	0.006	0.389	0.005	0.081
HS	0.000	0.000	0.007	0.001	0.009	0.001	0.033	0.014
Mexican peso								
CAViaR	0.000	0.051	0.190	0.060	0.004	0.900	0.263	0.124
EGARCH	0.000	0.000	0.000	0.000	0.040	0.017	0.012	0.001
HS	0.000	0.000	0.147	0.066	0.000	0.000	0.009	0.001
Argentine peso								
CAViaR					0.000	0.000	0.000	0.000
EGARCH					0.000	0.000	0.000	0.000
HS					0.000	0.000	0.052	0.065

Note: a minimum p-value is shown for each stock index and its national currency analysis for each UC and IND test. If the minimum p-value is greater than 10%, the null hypothesis cannot be rejected. If both tests are not rejected, the model is considered valid for the relevant index and the particular period under study.