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Operational Risk in the Settlement Process of the Mexican Stock Market: A Bayesian approach

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Abstract

This paper identifies and quantifies diverse operational-risk (OR) factors in the settlement process of the Mexican stock market through a Bayesian network (BN). The BN model is calibrated with data from events that occurred in the settlement process at the *Instituto de Depósito de Valores* (INDEVAL) from 2007 to 2010, and with additional information obtained from experts at the Institute. Unlike traditional methods, the BN model calibration uses both objective and subjective information sources to express the relationship between risk factors (cause and effect), strengthening its usefulness as shown in the comparative analysis carried out on BN and traditional approaches. It is important to mention that the proposed Bayesian approach is consistent in the sense of Artzner (1998).

Key words: operational risk, Bayesian analysis, Monte Carlo simulation.
JEL **Classification**: C11, C15, D81.

Introduction

The Bayesian approach is a feasible alternative for risk analysis in conditions of insufficient information. By construction, the Bayesian models incorporate initial information by means of an *a priori* probability distribution, which includes subjective information in decision making, such as expert opinion, analysts' judgments, or specialists' beliefs. This paper uses a Bayesian network (BN) model to examine the interrelation between operational risk[1] (OR) within the settlement process carried out by the *Instituto de Depósito de Valores* (INDEVAL) in Mexico.

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[1] Also known as operative risk.

The proposed BN model is calibrated with real data from events that occurred in the INDEVAL settlement process and from information obtained from the Institute's own experts[2] from 2007 to 2010.

Bayesian models incorporate uncertainty by means of a parametric probability distribution (sample model), and also allow the addition of initial subjective information by means of an *a priori* distribution, which can consider expert opinion or analysts' beliefs. The Bayesian approach, together with network topology, becomes relevant as an alternative for analyzing the administration of the OR from an economic and financial perspective.

The OR usually involves a small part of commercial banks' total annual losses; yet when an severe operational event occurs, sizable losses can accrue. For this reason, large-scale changes taking place in the banking industry throughout the world strive to improve policies and recommendations regarding operational risk.

Notably, specialized literature has various statistical techniques to identify and quantify the OR, which share a fundamental assumption regarding independence among risk events. Examples are Degen, Embrechts, and Lambrigger (2007), Moscadelli (2004), and Embrechts, Furrer, and Kaufmann (2003). Yet, as we can see in the work of Aquaro *et al.* (2009), Supatgiat, Kenyon, and Henssler (2006), Marcelo (2004), Neil, Márquez, and Fenton (2004), and Alexander (2002), there is a causal relationship among the OR factors.

In spite of research done by Reimer and Neu (2002; 2003), Kartik and Reimer (2007), Leippold (2003), Aquaro *et al.* (2009), Neil, Márquez, and Fenton (2004), and Alexander (2002), that addresses in general terms the application of BN in the administration of OR, no complete guide exists on how to classify the OR events, how to identify or quantify them, or how to calculate the economic capital in a consistent way.[3] This paper endeavors to close these gaps by, first, preparing information structures on OR events in such a way so as to allow the identification, quantification, and measurement of the OR; and, second, by changing the assumption of event independence in order to model the causal behavior of OR events more realistically. To do so, we examine the correlation

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[2] When reference is made herein to experts, they are experienced INDEVAL officials who are knowledgeable about the operation and administration of the business lines associated with the settlement process.

[3] To measure the maximum anticipated loss (or economic capital) due to OR, the Conditional Value at Risk (CVaR) is usually used.

among risk factors to develop a BN model that identifies and quantifies the OR of the settlement process in the Mexican stock exchange.

This paper is divided into five sections, in addition to this introduction. The first covers the typology and methods of calculating OR in line with Basel II (2001a). Then we analyze the theoretical framework that undergirds the development of the paper's argument, emphasizing the characteristics and advantages of the BNs. The following section analyzes INDEVAL's settlement process and the problems to be solved, as well as the scope of the application of the proposed methodology. Then a BN is developed based on an analysis of the risk factors associated with INDEVAL's security settlements procedure. Two networks are obtained, one for frequency, the other for severity. To quantify each network node and obtain the *a priori* probabilities, probability distributions are "adjusted" for those cases where there is historic information (2007-2010); if no such information exists, we use experts' opinion or judgment to obtain the corresponding probabilities. Once we have the *a priori* probabilities of the two networks, we then calculate the *a posteriori* probabilities by means of Bayesian inference algorithms, specifically using the junction-tree algorithm. In the final section we calculate the conditional operational risk of INDEVAL's settlement process by means of a Monte Carlo simulation with *a posteriori* distributions calculated for frequency and severity.

Types of operational risk

The most common definition of the concept of operational risk was established by the Basel Committee: "the risk of direct or indirect losses as a result of system failures, inadequate internal processes, human errors, and external events." This definition has an operational focus due to the fact that internal processes include both the procedure as such, as well as the internal processes. In sum, there are basically four dimensions of operational risk: human factors, systems, procedures, and external events.

Risk identification

The idea that OR can only occur in operations may be erroneous. This type of risk can occur anyplace or any time employees, systems, or procedures play a part in the daily work routine or where financial institutions are exposed to risks and external attacks.

Measuring the operational risk

The nature of methods used to quantify and measure operational risk varies from the simplest to the most complex methods, and among models that use just a single indicator to very sophisticated statistical models.

Methods to measure operational risk

The following is a brief description of the methods found in the literature on OR measurement (see, for example, Heinrich, 2006; Basel II, 2001b):

1) *Top-Down* single-indicator methods. This method was chosen by the Basel Committee as a first approximation in calculating operational risk. A lone indicator, such as the institution's total income or income volatility, or total expenditures, can be considered the total liability to be covered given a particular risk.
2) *Bottom-Up* methods include the judgment of an expert. The basis for an expert's analysis is a set of scenarios. Experts identify the risks and the probability of their occurrence.
3) Internal measurement. The Basel Committee proposed a method of internal measurement as a more advanced procedure to calculate the cost of regulatory capital.
4) Traditional statistical approach. Analogously to what occurred with the quantification methods for market risk and, more recently, credit risk, research has also made strides regarding methods of calculating operational risk. Yet, as opposed to market risk, it is very difficult to find a widely-accepted statistical method.
5) Causal models. As an alternative to traditional statistics, causal models have been proposed that assume independence between risk events; in other words, each event represents a random variable (discreet or continuous) with a conditional distribution function. For those events without a historical record or the quality of which is inadequate, the opinion or judgment of experts is sought in order to determine the conditional probabilities of the event. The tool for modeling this causality is the BN, grounded in Bayes' theorem and in network topology.

Theoretical framework

In this section we take up the theory underpinning this paper. We begin by discussing the conditional value at risk (CVaR) as a measure of "coherent" risk in the sense of Artzner *et al.* (1998). We then use the Bayesian approach to construct a BN, highlighting their advantages over the traditional approach to the study of OR.

Conditional value at risk (CVaR)

According to Panjer (2006), the CVaR or expected shortfall is an alternative measure of value at risk (VaR) that quantifies losses that can be found in distribution tails. It is defined as an expected loss for cases where the portfolio loss exceeds the value of the VaR.

If X denotes a random loss variable, the CVaR of X is a confidence level of $(1-p)\times 100\%$, expressed as $\mathrm{CVaR}p(X)$, which is the expected loss, given that total losses exceed the $100p$ quantile of the distribution of X. For arbitrary distributions we can write $\mathrm{CVaR}p(X)$ as:

$$\mathrm{CVaR}_p(X)=\mathrm{E}[X\mid X>x_p]=\frac{\int_{x_p}^{\infty} x\,\mathrm{d}F(x)}{1-F(x_p)}$$

where $F(x)$ is the accumulated distribution function of X. Further, for continuous distributions, we can use the density function to write the foregoing as:

$$\mathrm{CVaR}_p(X)=\mathrm{E}[X\mid X>x_p]=\frac{\int_{x_p}^{\infty} x\,f(x)dx}{1-F(x_p)}=\frac{\int_{p}^{1}\mathrm{VaR}_u(x)du}{1-p} \qquad [1]$$

Thus, the CVaR can be seen as the average of all VaR values above a confidence level p. In addition, CVaR can be written:

$$\mathrm{CVaR}_p(X)=\mathrm{E}[X\mid X>x_p]=x_p+\frac{\int_{x_p}^{\infty}(x-x_p)dF(x)}{1-F(x_p)}=\mathrm{VaR}_p(x)+e(x_p) \qquad [2]$$

where $e(x_p)$ is the average excess of the loss function.[4]

[4] For a detailed analysis of the non-coherence of VaR, see Venegas-Martínez (2006).

The Bayesian paradigm

In statistical analysis there are two philosophical paradigms: frequency and Bayesian. The fundamental difference between the two has to do with a definition of probability. Those favoring the frequency paradigm say that the probability of an event is the limit of its relative frequency in the long run; the Bayesians hold that probability is subjective, *i.e.*, a level of belief that is updated when new information is incorporated; subjective probability (belief) grounded in a knowledge base becomes the *a priori* probability; the *a posteriori* probability represents updated beliefs.

A Bayesian decision maker learns and revises his/her beliefs based on new available information. From a Bayesian point of view, probabilities are interpreted as levels of belief. Therefore, the Bayesian learning process consists of revising and updating probabilities. Bayes' theorem is the formal way to put this into practice.[5]

Bayes' theorem

Bayes' theorem is a rule that can be used to update beliefs based on new information (for example, observed data). If we denote evidence with E and assume that an expert believes that it can be associated with a probability $P(E)$, Bayes' theorem says that after observing data (D), the beliefs about E are adjusted according to the following expression:

$$\frac{P(D|E)P(E)}{P(D)} \qquad [3]$$

where $P(D \mid E)$ is the conditional probability of the data, given that the *a priori* evidence D is certain, and $P(D)$ is the unconditional probability of the data, $P(D) > 0$. This can also be expressed as:

$$P(D) = P(D \mid E)P(E) + P(D \mid E^C)P(E^C)$$

The probability of E, before receiving data $P(E)$, is called *a priori* probability; once updated, $P(E \mid D)$ is called the *a posteriori* probability.

5 To review Bayes' theorem, see, for example, Zellner (1971).

We can rewrite the continuous form of Bayes' theorem as follows:

$$P(\theta \mid \mathbf{y}) \propto L(\theta \mid \mathbf{y})\pi(\theta) \qquad [4]$$

where θ is an unknown parameter to be estimated; $\mathbf{y}$ is a vector of registered observations; $\pi(\theta)$ is an *a priori* distribution that depends on one or more parameters, called hyper-parameters; $L(\theta \mid \mathbf{y})$ is a likelihood function for θ, and $P(\theta \mid \mathbf{y})$ is the *a posteriori* distribution of θ (updating the *a priori* distribution). Two questions arise from the above: how to translate the *a priori* information into analytic form, $\pi(\theta)$, and how sensitive is the *a posteriori* inference to the *a priori* selection. These questions have been a rich topic of interest in Bayesian literature (see Ferguson, 1973).

Bayesian inference

The *a posteriori* distribution of the parameter or vector θ, given available information $\mathbf{y}$ and denoted by $P(\theta \mid \mathbf{y})$, is obtained by applying Bayes' theorem. It is a combination of data and the *a priori* distribution, while the *a posteriori* distribution has relevant information regarding the unknown parameter.

Bayesian networks

A Bayesian network is a graph that represents the domain of the decision variables, their quantitative and qualitative relationships, and their probability parameters. Worth noting is the quantitative aspect of the BNs, since they allow subjective elements to be incorporated, such as expert opinion and probabilities based on statistical data. Each node in a BN is associated with a set of probability tables. The nodes represent the variables of interest, either discrete or continuous. According to Pearl (2000) a causal network is a BN with an additional property: "parent" nodes are directed causes.

Theory of Bayesian networks

According to Jensen (1996), the mathematical definition of a Bayesian network consists of:

1) A set of variables connected by a set of directed links.
2) Each variable is associated with a finite set of mutually exclusive states.
3) Variables, together with directed links make an acyclical directed graph (ADG).

4) For each variable A with parents $B_1,\ldots,B_n$, there is an associated probability defined by $P(A \mid B_1,\ldots,B_n)$. Note that if A does not have parent nodes, the probability of $P(A)$ is unconditional.

Let $X = \{x_1,x_2,\ldots,x_n\}$ be a random variable with a joint distribution function defined by $P(X) = P(x_1,x_2,\ldots,x_n)$. Bayesian networks give a compact representation of $P(X)$ by factoring a joint distribution in a local conditional distribution for each variable, given its parent nodes. Let $pa(x_i)$ be the set of values taken by the parent nodes of variable x; then the total joint distribution would be given by:

$$P(x_1,x_2,\ldots,x_n) = \pi[x_i \mid pa(x_i)]$$

Algorithms for calculating inference in Bayesian networks

A Bayesian network is basically used for inference by calculating conditional probabilities, given current available information for each node (beliefs). There are two classes of algorithms for the inference process: the first generates an exact solution, and the second produces an approximate solution with high probability. Among algorithms with exact inference we have, for example, polytree, clique tree, junction tree, algorithms variable elimination and the Pear method.

Construction of a Bayesian network for the settlement process in the Mexican stock market

The first step in building a BN is defining the domain of the problem by specifying its purpose. In what follows we identify the variables or important nodes in the domain of the problem. Then the interrelation between nodes or variables is graphed. The resulting model should be validated by experts in the matter. Should there be disagreement among them, we return to one of the previous steps until a consensus is reached. The final three steps are: incorporate expert opinion (referred to as the quantification of the network), create feasible scenarios with the network (application of networks), and adjust the estimations over time (network maintenance).

Problems

The main problems faced by a risk administrator using BN are: how to implement a Bayesian network, how to model its structure, how to quantify it, how to use subjective data (from experts) or objective data (statistics), or both, what instruments should be used to obtain the best results, and how to validate the model. Answers to these questions will be taken up in the application of the Bayesian model.

The principal objective of the application consists of preparing a guide for implementing a BN in order to administer the operational risk in the settlement process of the Mexican stock market. Similarly, we hope to generate a consistent measure of necessary economic capital to address losses stemming from operational-risk events.

Scope of the application

This case study focuses on an analysis of the match, pre-settlement, compensation, and settlement sub-processes, which are an integral part of INDEVAL's complete compensation process. Once the risk factors associated with each sub-process have been identified, the nodes that will be part of the Bayesian network are defined. These are random variables that can be discrete or continuous and have associated probability distributions.

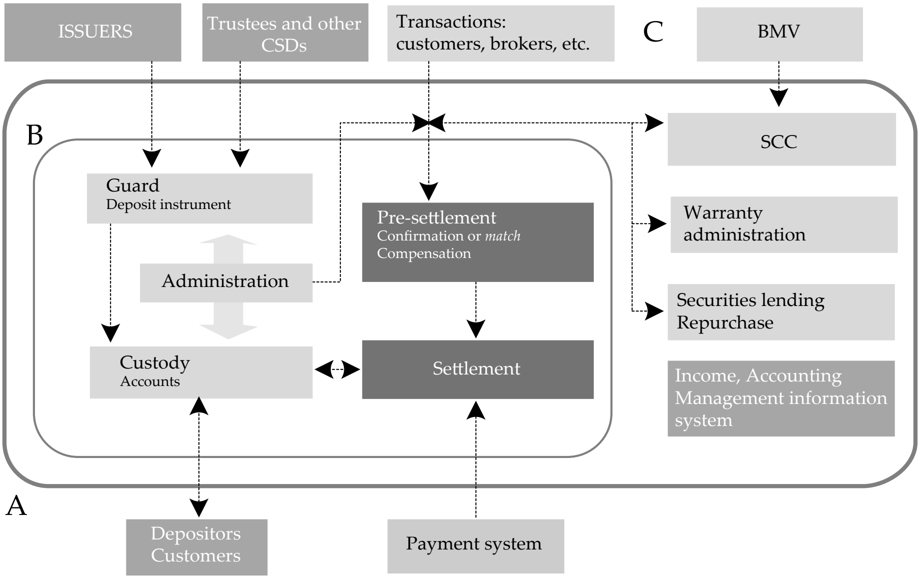
Should historical data related to the nodes (random variables) be available, they are adjusted with a distribution function; otherwise, expert guidance is sought to determine probabilities of occurrence or the parameter of some known probability function. Known data are daily readings and cover 2007-2010. The calculation of the maximum expected loss will be for a day. It is worthwhile mentioning that INDEVAL has other substantive processes, such as custody and securities lending. They are not, however, studied here.

The next section will analyze INDEVAL's settlement process flow (securities settlement procedure), which will allow us to identify the risk factors associated with the operation that, in turn, enables us to define the nodes that will be part of the BN.

Securities settlement procedure

As previously mentioned, the securities settlement procedure has four sub-processes: match, pre-settlement, compensation and settlement. A settlement operation contains credit or debit orders in securities or cash accounts. These orders are known as "actions". Graph 1 summarizes INDEVAL's securities settlement procedure.

Graph 1
Flow of INDEVAL's securities settlement procedure



Source: INDEVAL.

Model construction and quantification

Constructing a network is a two-step process: 1) creating the model's structure, taken up in the following two points of this section; and 2) quantification of the network, covered in the third point in this section.

Risk, process mapping, and node identification

To identify the risk factors, the overall process is divided into three sub-processes: match, pre-settlement, and settlement. In each sub-process, we define activities, possible associated risks, and factors that make risks occur. For example, inputting activities include: transaction request, registration, and transmitting instructions. These activities depend on the front-office's procedures and on the operational personnel. Among the possible risks are: erroneous registration, transmission system failure, or error in the previous entry. Risk factors would include transaction volume, system availability, and the level of training afforded to the inputting personnel. A complete description of the sub-processes, activities, and risks is found in Table 1.

Table 1
Mapping of process and risks

	Match	*Pre-settlement*	*Settlement*	*Post-settlement*
Activities	Undertake transaction	Receive transaction order	Prepare transfer documents	Foresee movement of cash
	Transaction registration	Validate transaction	Transmit to the central bank and to area in charge of account transactions	Receive settlement information from area in charge of handling accounts
	Transmit transaction instruction	Compensation		Conciliate and compare expected cash movements with the actual transaction
		Transmit transaction		
Office	Front-office system	Back-office system	Payment system	
	Human resource	Manual operation	Accounts administration system	
		Human resource	Human resource	
Risk factors	Error in transaction registration	Validation error	Delayed payments	Reconciliation errors
	Transmission system failure	Confirmation error	Duplicate payments	Untimely accounting
	Entry made outside transmission time		Incorrect settlements	Insufficient funds or overdraft
			Unauthorized payments (buy/sell)	
			Lack of payment	
Risk factors	Transaction volume	Validation or confirmation method (manual/automatic)	Incorrect instructions	Reconciliation method (manual or automatic)
	System availability	System availability	Validation errors	System down time
		Type of transaction	Confirmation errors (settlement before confirmation)	External errors
			System down time	
			Same day/other day transaction	
			Registration after transaction time	
			Gross or net settlement	

Source: Compiled by authors.

The mapping of processes and risks generates a list of activities, offices, risks, risk factors, and key risk indicators that are all "candidates" to be used as nodes in building a Bayesian network.

Structure of the Bayesian network

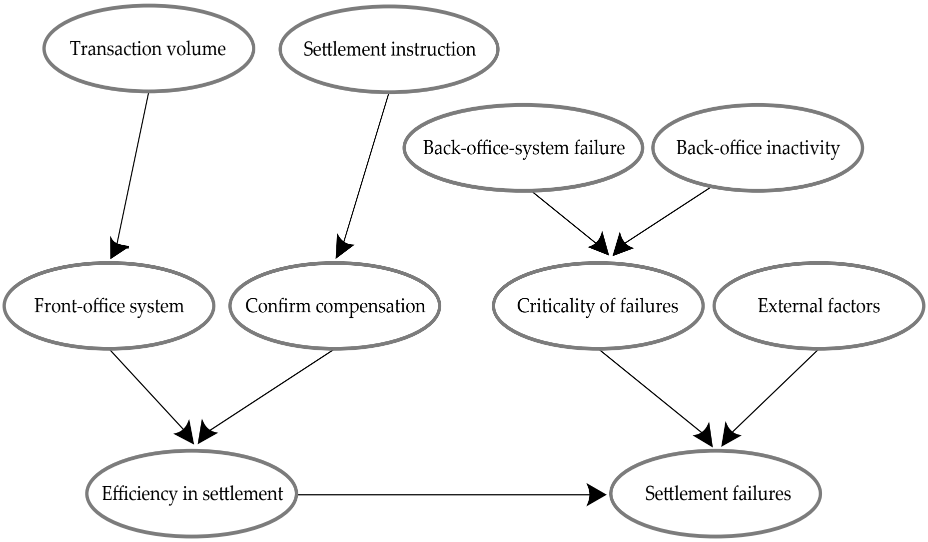
The selected nodes are connected with directed links (arrows) that make up a structure showing the dependency or causal relationship between them.

The network of the settlement process is divided into two networks: one to model frequency and the other for severity, a step that facilitates their analysis. Once results have been obtained, these are "added" separately through a Monte Carlo simulation in order to obtain the expected loss in the settlement process.

Frequency

The complete frequency network appears in Graph 2, which is generated from the main elements of the settlement process, as detailed in the chart of processes and risks.

Graph **2**
Failure frequency network

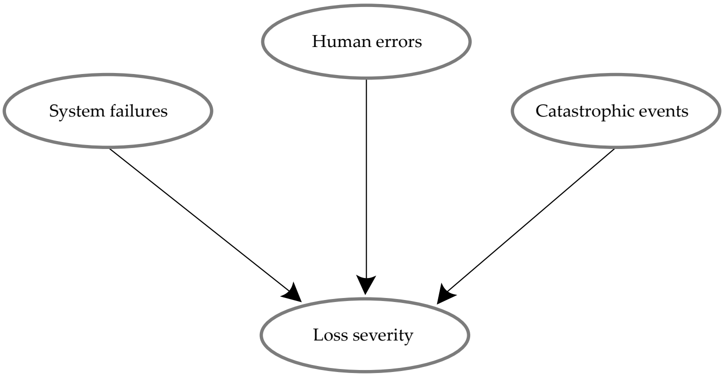


Source: prepared by the authors.

Severity

The severity network is shown in Graph 3. The network is made up of four nodes, but requires an important amount of probabilities. The node labeled "severity of loss" is the monetary loss created by a failure in the settlement of a position; the other nodes are considered to be information variables.

Graph **3**
Failure severity network



Source: prepared by the authors.

What follows is a description of the characteristics of each node of the severity and frequency nodes, respectively.

Table **2**
Severity network nodes

Node	*Description*	*States*
Systems failure	Failures in the INDEVAL settlement procedure	< 30 000 30 000 to 50 000 > 50 000
Human errors	Human errors that result in losses	< 5 000 5 000 to 20 000 > 20 000
Catastrophic events	External events such as demonstrations, threats, among others	< 50 000 50 000 to 100 000 > 100 000
Severity of losses	Expected loss due to operational risk events	< 40 000 40 000 to 100 000 > 100 000

Source: prepared by the authors.

Table 3
Frequency network nodes

Node name	*Description of node*	*States*
Transaction volume	Transaction volume by period	< 40 000 40 000-100 000 >100 000
Front-office system	Availability and proper working of front-office system	Working adequately (available and working correctly) Working poorly (system errors or slowness) Not available
Back-office system down time	Includes down time before a currency is disconnected	< 5 minutes 5-30 minutes > 30 minutes
Back-office-system failure	The time of the back-office failure	Not critical Critical
Settlement instruction	Refers to the settlement instruction	Yes Not
Confirm compensation	Confirm compensation before settling transaction. Note that if the settlement instruction is given, compensation is automatic; otherwise, it is done manually with the risk of associated errors	Not confirmed (not sent, sent without feedback, etc.) Undertaken incorrectly (manually, for example, by phone) Undertaken correctly
Efficiency in settlement instructions	Refers to the percentage of correct instructions to total instructions in a given period	Excellent 100% Average 98-100% Poor < 98%
Criticality of failures	Refers to the level of criticality of failures	Low Medium High
External factors	Number of external events impossible to foresee or administer	0,1,2,3,…,17
Settlement failures	Number of failures in the settlement process in a given time (delays, incorrect payments, poorly channeled payments, non-payments, duplicate payments)	0,1,2,3,…,17

Source: prepared by the authors.

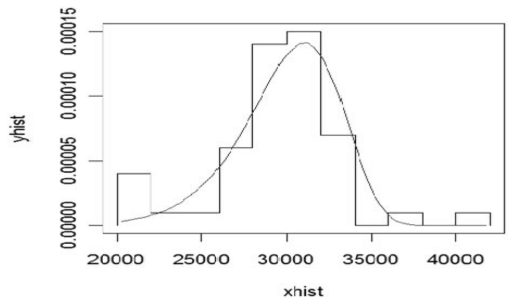
Quantification of the BN for the stock market

To quantify the Bayesian networks mentioned in the previous section, we used both objective and subjective data; yet available historical data are scarce or not easily codified for use within the network. Thus, we will use mainly subjective data in the frequency network. Insofar as the severity network is concerned, it is quantified with statistical (objective) data. In what follows we describe the tools or techniques used to obtain, codify, and quantify the data.

Statistical analysis of the frequency network

In this section, we analyze each frequency network node; in the case of nodes with available historic information, we adjust[6] several distributions until the best is found in line with the χ^2 statistical test; then we calculate the required probabilities. When lacking sufficient data, we turn to experts for information. The node labeled "transaction volume" has the following frequency distribution and adjusted Weibull density.

Graph **4**
Adjusted Weibull
(transaction volume)



Source: prepared by the authors.

[6] Adjusting a distribution consists of finding a mathematical function that correctly represents a statistical variable. Steps for the adjustment: 1) hypothesis of the model; 2) parameter estimates; 3) evaluation of quality of adjustment, and 4) statistical test regarding goodness of fit. Here we used the *R* statistical language: first we graphed the frequency distribution of real data so as to propose a distribution model; we then undertook various estimations to find the best parameter. We used the χ^2 test to determine statistically the goodness of fit. A p-value > 0.05 indicates a good fit.

We thus calculate a probability table for this frequency-network node, which also becomes the *a priori* probabilities.

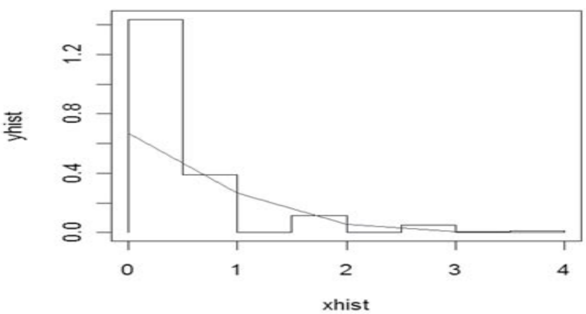
Table 4
Transaction volume node probabilities

Transaction volume	*Probability*
< 25 000	0.065
25 000 to 35 000	0.913
> 35 000	0.020

Source: prepared by the authors.

With regards to the node labeled "front-office system", these are data associated with risk derived from activities in the first level of the settlement process, such as input by clients, the taking of a particular position, among others. In Graph 5, we see the frequency distribution and the adjusted Poisson distribution with a $\lambda = 0.4$ parameter.[7]

Graph **5**
Adjusted Poisson
(Front-office failure)



Source: prepared by the authors.

The preceding graph shows the distribution of the number of failures in the front-office system; results show small probabilities that there will be more than

[7] The Poisson distribution has two important properties: the first is given by the following theorem: if $N_1,\ldots,N_n$ is a Poisson variable with parameters $\lambda_1,\ldots,\lambda_n$, then $N = N_1 + \ldots + N_n$ has a Poisson distribution with $\lambda_1 + \ldots + \lambda_n$ parameters. The second characteristic is particularly useful in modeling operational risk events. It assumes that the number of losses in a fixed period of time follows a Poisson distribution; further, it also assumes that losses can be classified in m distinct types.

one failure per day. Nonetheless, what we should calculate are the conditional probabilities that this system is indeed working, working poorly, or not working at all. Thus, considering previous results and expert knowledge, we calculate a table of conditional probability for this frequency-network node, which constitutes the *a priori* probabilities.

Table **5**
Conditional probabilities of the front-office system node

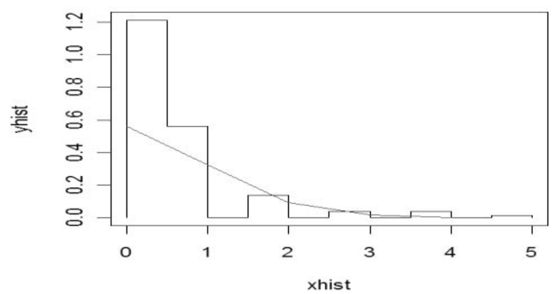
Transaction volume	*< 25 000*	*25 000 to 35 000*	*> 35 000*
Working	0.85	0.7	0.6
Working poorly	0.1	0.2	0.25
Not working	0.05	0.1	0.15

Source: prepared by the authors with information provided by experts.

Given that the transaction volume was less than 25 000 operations, an 85% probability exists that the front-office system works, 10% that it works incorrectly, and 5% that it does not work at all. The remaining conditional probabilities have a similar interpretation.

Regarding the node labeled "back-office-system failure", the following graph show its frequency distribution and adjusted Poisson probabilities.

Graph **6**
Adjusted Poisson
(*back-office failure*)



Source: prepared by the authors.

The preceding analysis calculates the probability function of the number of daily failures of the back-office system, which indicates a small possibility that the system will fail more than once. In line with the frequency network, we are interested in estimating the probabilities that the back-office system will fail in a critical or non-critical form. So, considering the previous results and expert knowledge regarding the criticality of failures of the INDEVAL procedure, we estimated the following *a priori* probabilities for the back-office-failure node.

Table **6**

Probabilities of the back-office system failure node

Back-office failure	*Probability*
Critical	0.04
Not critical	0.96

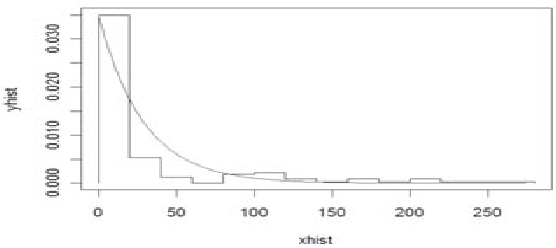
Source: prepared by the authors with information provided by experts.

With regards to the node labeled "back-office-system inactivity", the following graph shows its frequency distribution and the adjusted exponential density function.[8]

Graph **7**

Adjusted exponential

(back-office-inactivity time)



Source: prepared by the authors.

The probabilities associated with the states of this frequency-network node are calculated, which make up the *a prioriti* probabilities.

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[8] The exponential function is the only one with a continuous distribution and a constant failure rate, $h(x) = 1/\theta$, and an excess conditional expected loss, $e_d(x) = \theta$, which is also constant. Thus, the excess loss does not depend on the established threshold.

Table 7
Probabilities of the back-office-inactivity node

Length of transaction	*Probability*
< 25 minutes	0.58
25 to 120 minutes	0.40
> 120 minutes	0.02

Source: prepared by the authors.

For the remaining nodes that make up the frequency network, no information registered in a data base exists. Instead, experts' judgments and beliefs allowed us to obtain the probabilities associated with each state in each node. The process began by interviewing those responsible for registering operational risk events. Then a second review was carried out by those involved in the settlement process, in order to arrive at a consensus regarding the *a priori* probability distributions. The results of this process are summarized in the following probability tables.

Table 8
Probabilities of the settlement instruction mode

Settlement instruction	*Probability*
Yes	0.95
No	0.05

Source: prepared by the authors with information from experts.

Table 9
Conditional probabilities of the compensation-confirmation node

Settlement instruction	*Yes*	*No*
Undertaken correctly	0.89	0.85
Undertaken incorrectly	0.01	0.07
Not confirmed	0.10	0.08

Source: prepared by the authors with information provided by experts.

Table 10
Conditional probabilities of the settlement-efficiency node

Confirm compensation	*Correctly undertaken*		
Front-office system	Works	Works poorly	Does not work
Excellent 100%	0.85	0.82	0.78
Average 98%	0.10	0.13	0.17
Poor < 98%	0.05	0.05	0.05
	Undertaken incorrectly		
Front-office system	Works	Works poorly	Does not work
Excellent 100%	0.8	0.78	0.75
Average 98%	0.15	0.17	0.25
Poor < 98%	0.05	0.05	0.0
	Not confirmed		
Front-office system	Works	Works poorly	Does not work
Excellent 100%	0.79	0.78	0.75
Average 98%	0.15	0.16	0.20
Poor < 98%	0.06	0.06	0.05

Source: prepared by the authors with information provided by experts.

A conditional probability exists, given that a compensation confirmation was correctly transmitted and the front-office system works properly: there is a 85% probability that there will be 100% efficiency in the settlement; 10% that this will be between 98-99%, and 5% that it will be less than 98%. The other conditional probabilities are read in a similar way. The following node measures the impact of potential failures in the settlement process.

Table 11
Conditional probabilities of the failure-criticality node

Low inactivity	*< 25 minutos*		*25 a 120 minutos*		*> 120 minutos*	
Back-office-system failure	Critical	Not critical	Critical	Not critical	Critical	Not critical
Low	0.05	0.05	0.06	0.05	0.0	0.05
Medium	0.10	0.25	0.04	0.20	0.05	0.15
High	0.85	0.70	0.90	0.75	0.95	0.80

Source: prepared by the authors with information from experts.

A conditional probability exists, given that there is back-office inactivity under 25 minutes and a critical failure of the system is present. There is a 5% probability that the criticality will be low, 10% that it will be average, and 85% that it will be high. The other probabilities are read in a similar manner.

Table **12**
Probability of the external-factors node

External factors	*Probability*
1	0.489
2	0.39
3	0.05
4	0.04
5	0.02
6	0.01
> 6	0.001

Source: prepared by the authors with information from experts.

There is a 49% probability that an external operational-risk event will occur, 39% that two events will occur, and 12% that more than two such risk events will occur.

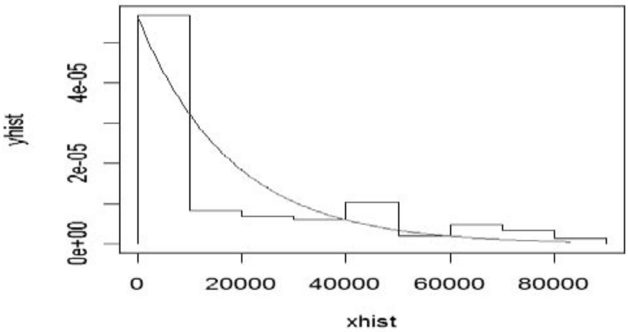
Finally, the settlement-failures target node works on the assumption of a Poisson probability function with a $\lambda = 0.6$ parameter; this assumption is consistent with financial practices and operational risk studies that show that the number of failures usually conforms to a Poisson distribution or a negative binomial. Yet the latter is very disperse in processes where expert opinion is included for parameter estimation. To estimate the value of the λ parameter, expert opinion was sought and complemented with the results of the analysis of the front-office-system-failure and the back-office-system-failure nodes, which are a fundamental part of the settlement process. Further, these nodes will be used to analyze the system's sensitivity.

Statistical analysis of the severity network

In this section we analyze every node in the severity network. For nodes with available historical information, we adjust the best probability distribution in line with the χ^2 test and calculate the probabilities as needed. Expert information is sought when sufficient data in unavailable.

The systems-failure node has the following frequency distribution and adjusted exponential density for losses caused by systems failures.

Graph 8
Adjusted exponential
(systems loss)



Source: prepared by the authors.

We then calculate a probability table for this node in the severity network, showing the *a priori* probabilities.

Table 13
Probabilities of the systems-failure node

System failure	*Probability*
< 30000	0.814
30000 to 50000	0.125
> 50000	0.060

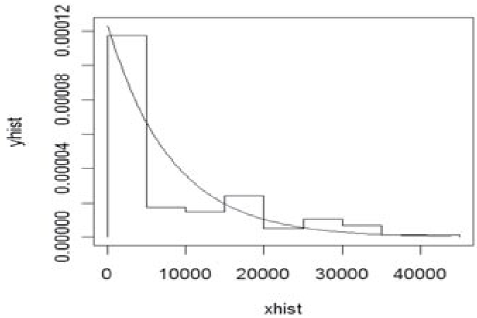
Source: prepared by the authors.

From the table we can see that an 82% probability exists that less than $30 000 will be lost due to systems failures, a 12% chance that between $30 000-50 000 will be lost, and a 6% chance that the loss will be above $50 000.

The human-errors node has the following frequency distribution and adjusted exponential density for losses due to human errors.

We calculate the probabilities of this severity network node, which become the *a priori* probabilities.

Graph 9
Adjusted exponential
(human-error losses)



Source: prepared by the authors.

Table **14**
Probabilities of the human-errors node

Human errors	*Probability*
< 5000	0.460
5000 to 20000	0.454
> 20000	0.084

Source: prepared by the authors.

From the table we see that a 46% probability exists that less than $5 000 will be lost due to human errors, a 45% chance that between $5 000-20 000 pesos will be lost, and a 9% chance that the loss will be greater than $20 000.

There is no information recorded in data bases for the catastrophic event node, so we obtained experts' judgments and beliefs for the possibilities associated for each state.

Table **15**
Probabilities for the catastrophic-events node

Catastrophic events	*Probability*
< 5 000	0.053
5 000 to 100 000	0.253
> 100 000	0.692

Source: prepared by the authors with information from experts.

According to experts, there is a 5% probability that less than $5 000 will be lost due to catastrophic events, a 25% probability that between $5 000 and $10 000 will be lost, and a 70% chance that the loss will be above $100 000.

Finally, the severity-of-loss goal node represents the sum of losses associated with the system-failure, human-error, and catastrophic-event nodes. To calculate the conditional probability table, we used an exponential distribution function with a parameter equal to the average of the previous nodes. The assumption of exponential distribution is consistent with adjusted exponential distributions for the system-failure and human-error nodes. In the next section, we generate *a posteriori* probabilities, using Bayesian inference techniques.

A posteriori probabilities

Once each node (continuous or discrete random variables) of the frequency and severity networks has been analyzed and the corresponding probability distribution functions assigned, we generate the *a posteriori* probabilities using inference techniques for Bayesian networks. The so-called junction tree, one of several exact solution algorithms, is used to undertake the inference, given that it reduces the frequency and severity networks to their minimum expression, thus avoiding cycles and therefore optimizing processing time. For detailed information on algorithms, see Guo and Hsu (2002). The *a posteriori* probabilities for the frequency network nodes with at least one parent[9] are shown in Graph 10.

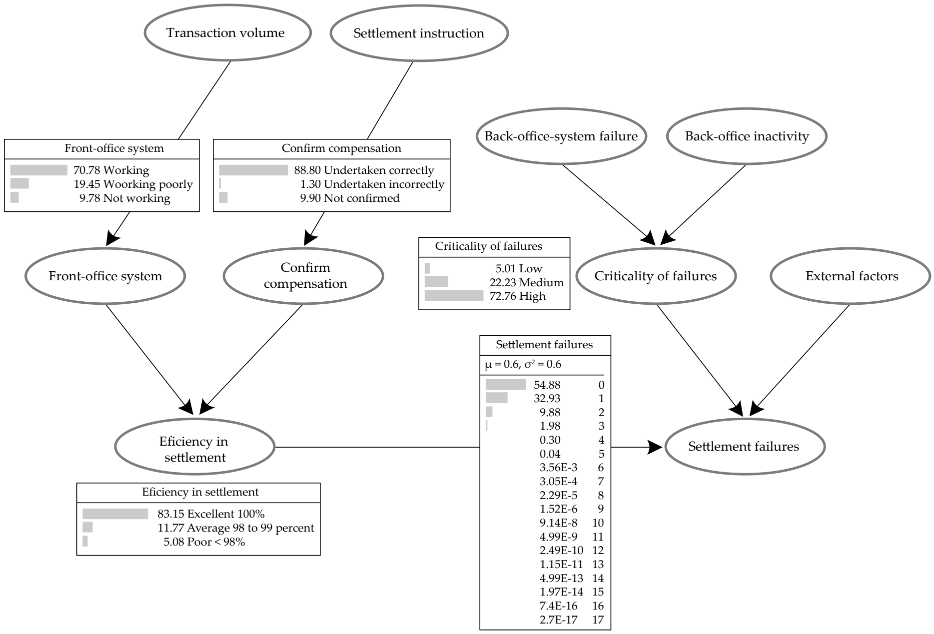
Results from the front-office-system node show that a 71% probability exists that the operations requested via the securities settlement procedure will be recorded and processed without a problem, a 20% probability that some minor problem will occur without delaying the process, and a 9% probability that the system will not work. These calculated probabilities are conditioned by actual transaction volume.

Regarding the confirm-compensation node, there is a 89% probability that the settlement order will be correctly confirmed, a 1% probability that it will be incorrectly confirmed, and a 10% probability that it will not be confirmed. These are conditioned to the existence of a settlement instruction.

Regarding the criticality of the failures of the securities settlement procedure as registered by the failure-criticality node, there is a 5% chance that it will be

[9] Nodes without a parent maintain the *a priori* probabilities.

Graph 10
A posteriori ***probabilities for the frequency network***



Source: prepared by the authors.

a low-level failure, a 22% chance that it will be a mid-level failure, and a 73% chance that it will be a high-level failure. These percentages are conditioned to the failures or inactivity of the back-office. The level of criticality is a key variable in the system, because if it is high, a systemic risk may occur, *i.e.*, it could put the entire national payments procedure at risk.

The settlement-efficiency node has an 83% probability that operations will be settled correctly, a 12% probability that they will be acceptable (average), and a 5% probability that they will be settled deficiently (poorly); probabilities are conditioned to the functioning of the front-office and to the way that the compensation is confirmed.

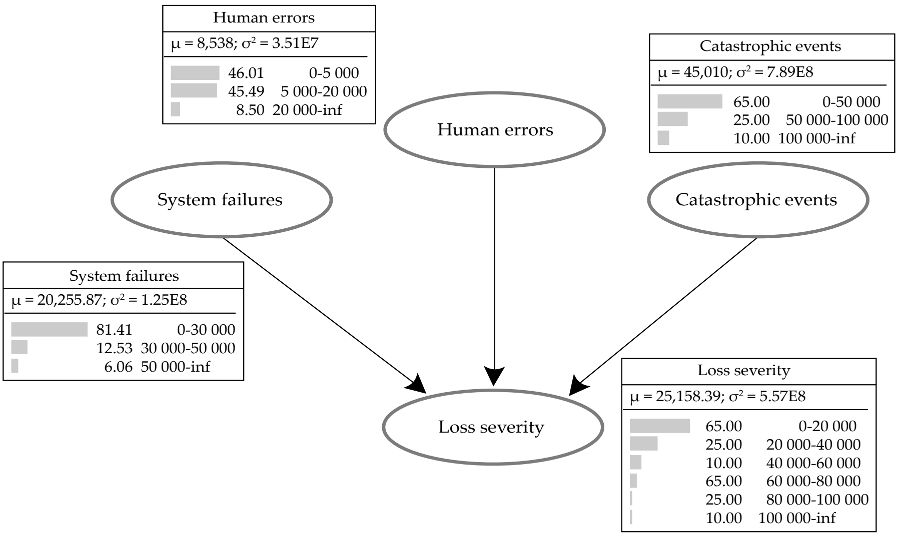
Finally, the probability distribution of the settlement-failures node-of-interest show a 55.88% chance that no failure will occur, 32.93% that one failure will occur, 9.88% that two failures will occur, 1.98% that three failures will occur, and 0.3% that there will be four or more failures. The probabilities are conditioned on external risk factors, criticality of settlement-system failures, and on settlement efficiency.

To calculate node-of-interest probabilities, we used a Poisson with a $\lambda = 0.6$ parameter; this value was selected considering the results of the frequency analysis of the front-office-failure and back-office-failure nodes. In addition, it is consistent with empirical evidence that the frequency of operational-risk events has an adequate adjustment under this distribution (see Svetlozar *et al.*, 2008).

We obtain the following *a posteriori* probabilities for the severity network.

Graph **11**

A posteriori probabilities for the severity network



Source: prepared by the authors.

Losses due to human error run 8 538 pesos per day on average. Regarding losses due to catastrophic events, including demonstrations, floods, and the like, on average they amount to 45 010 pesos per day. The reason for the relatively low loss has to do with the security-settlement procedure's high levels of security and availability, including an alternative headquarters should this type of event occur.

Regarding systems failures, on average there is a daily loss of \$20 255. The probability distribution of the loss-severity node of interest shows a 59.9% chance that the loss will be between 0 and 20 000 pesos; a 21% chance that it will be between 20 000 and 40 000; a 9.2% chance of losses between 40 000 and 60 000 pesos; a 4.4% chance of losses between 60 000 and 80 000 pesos;

a 2.3% chance that they will be between 80 000 and 100 000 pesos; and a 3% chance that losses will surpass $100 000 pesos in a single day.

To calculate the *a posteriori* node-of-interest probabilities, we used an exponential density with a parameter equal to the average of the losses due to human error, systems failures, and catastrophic events. The exponential distribution is consistent with the adjustment of the probability functions estimated in the statistical analysis of the aforementioned severity network.

Sensitivity analysis

In order to measure the model's sensitivity to changes in *a priori* probabilities, the Poisson distribution functions for the front-office-system and back-office-system nodes in the Bayesian networks were substituted for the exponential distribution functions with $\lambda = 1.5$ and $\lambda = 1.2$ parameters, respectively. The following tables contain the corresponding conditional probabilities.

Table **16**
Conditional probabilities of the front-office-system node with an exponential distribution function

Transaction volume	*< 25 000*	*25 000 to 35 000*	*> 35 000*
Working	0.9	0.8	0.7
Working poorly	0.05	0.1	0.20
Not working	0.05	0.1	0.1

Source: prepared by the authors with information provided by experts.

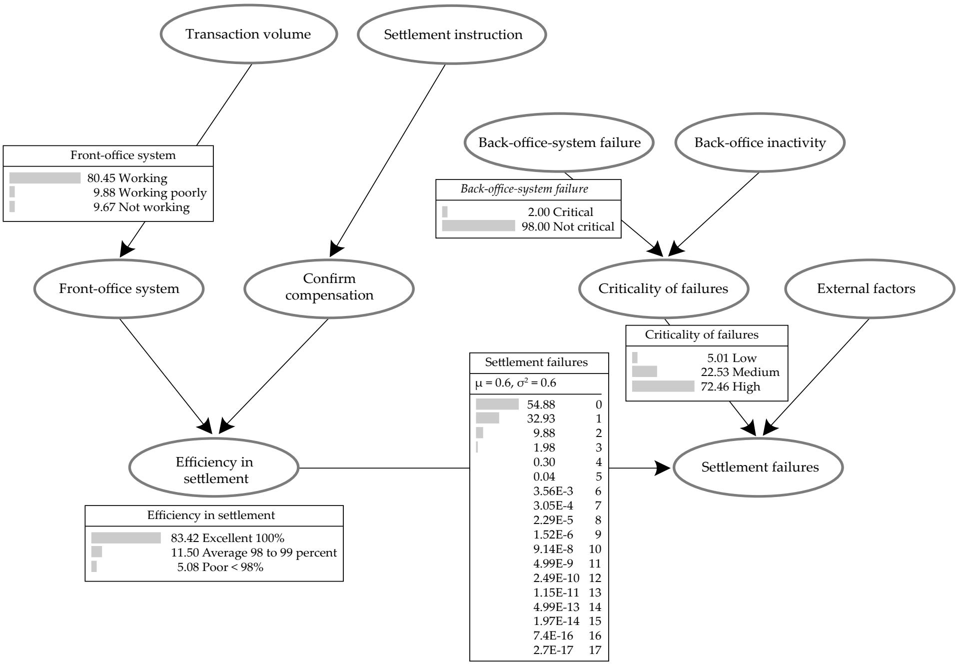
Table **17**
Probabilities of the back-office-system node with an exponential distribution function

Back-office failure	*Probability*
Critical	0.02
Not critical	0.98

Source: prepared by the authors with information provided by experts.

Substituting the *a priori* probabilities for the front-office and back-office systems and using the same Bayesian inference algorithm, we again calculated the *a posteriori* probabilities and obtained the following results.

Graph 12
A posteriori *probabilities for the frequency network with different* **a priori** *probabilities*



Source: prepared by the authors.

Comparing the original *a posteriori* probabilities (see Graph 10) with those calculated in this section (see Graph 12), we can conclude the following:

1) Due to the change in the *a priori* probability distribution function of the front-office-system node, the *a posteriori* probabilities change in the front-office-system and settlement-efficiency nodes.
2) The change in the *a priori* probability function of the back-office-system node has no effect on the *a posteriori* probabilities of the failure-criticality node.
3) The changes in the *a priori* probabilities of the front-office and back-office systems nodes have no effect on the *a posteriori* probabilities of the settlement-failure goal node. This shows that the Bayesian network constructed after a determined number of iterations converges at the same distribution goal with different *a priori* distributions, in spite of the fact that intermediate nodes can have some change in their probability distribution. Nonetheless, we cannot conclude that any *a priori* distribution will have the same convergence and results.

Calculating the operational value at risk (OpVaR)

After undertaking the Bayesian inference to obtain the *a posteriori* probability distribution of the frequency and loss severity, by means of a Monte Carlo simulation process (10 000 simulations), we integrate both distributions to generate a potential loss distribution (using a Poisson with a $\lambda = 0.6$ parameter for the frequency, and an exponential with a $r = 25{,}158$ parameter for severity) in the stock market settlement process.[10]

To calculate the operational value at risk (OpVar), we arranged in descending order values obtained for expected losses and calculated the corresponding percentiles. Table 18 shows results with confidence levels above 98.9 percent.

Thus we obtain a maximum expected loss of 128 047 pesos per day with a confidence level of 99%. To calculate the conditional VaR, we obtain an average of losses above a maximum expected loss and add it to the calculated OpVaR. Therefore, the CVaR for the operational risk of the Mexican stock market's settlement process is 280 226 pesos per day.

Bayesian model validation

To validate results from the Bayesian model, we estimated the probability distribution for frequency and severity through traditional models. Then, through a Monte Carlo simulation, we integrated both distributions to obtain a expected losses distribution. Lastly, we calculated the operational risk with the estimated loss distribution in the traditional way and compared results with those from the Bayesian model.

Traditional frequency analysis

We consider the number of failures that occur daily in INDEVAL's securities settlement process and adjust it with a Poisson distribution with a $\lambda = 0.6$ parameter, as seen in the following graph.

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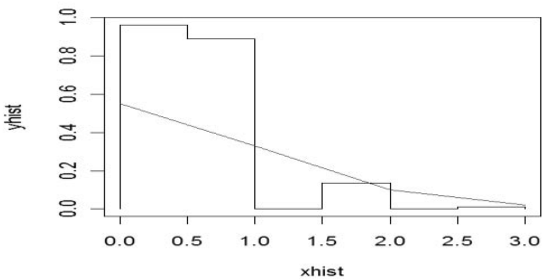
[10] Readers interested in reviewing the simulation results are asked to contact the authors by email.

Table 18
Bayesian model percentiles

Position	*Loss*	*Percentage*	*Position*	*Loss*	*Percentage*	*Position*	*Loss*	*Percentage*
2103	239 313.12	100.00	222	154 194.08	99.60	6924	136 256.33	99.20
8976	213 152.51	99.90	8884	153 787.95	99.60	3426	136 083.84	99.20
27	203 018.88	99.90	3705	153 212.33	99.50	7530	135 607.28	99.20
5901	199 950.08	99.90	311	152 288.94	99.50	1097	135 571.80	99.20
7229	187 705.70	99.90	5326	150 985.23	99.50	6965	134 989.96	99.10
6455	187 524.04	99.90	919	150 384.73	99.50	8068	134 112.84	99.10
6135	185 465.14	99.90	486	149 129.03	99.50	7880	134 013.37	99.10
6546	185 307.37	99.90	7634	149 126.07	99.50	4862	133 833.83	99.10
2237	183 634.97	99.90	993	149 063.51	99.50	4047	133 382.28	99.10
8616	181 100.46	99.90	4381	148 931.94	99.50	4384	132 738.58	99.10
4382	180 380.45	99.80	2154	148 302.14	99.50	2943	132 196.58	99.10
4177	172 486.07	99.80	9082	147 371.88	99.50	91	132 023.54	99.10
6173	171 815.49	99.80	7430	146 304.00	99.40	2361	131 929.16	99.10
6151	171 330.69	99.80	3975	145 729.86	99.40	5424	131 685.31	99.10
3424	170 222.34	99.80	8251	145 186.08	99.40	7030	131 556.15	99.00
7914	169 697.42	99.80	5747	145 178.81	99.40	1555	130 656.72	99.00
8748	169 545.90	99.80	6368	144 770.91	99.40	944	130 431.24	99.00
1318	167 555.95	99.80	7608	144 386.21	99.40	3016	130 163.66	99.00
2521	167 102.30	99.80	1910	144 097.05	99.40	3997	129 729.51	99.00
3124	165 663.06	99.80	6602	143 846.86	99.40	6859	129 009.59	99.00
2749	165 402.39	99.70	6622	143 639.32	99.40	4685	128 960.09	99.00
909	164 877.47	99.70	5967	143 256.44	99.40	170	128 293.91	99.00
3019	163 804.96	99.70	6482	142 320.25	99.30	5396	128 155.89	99.00
7742	162 948.18	99.70	7440	141 924.36	99.30	**1452**	**128 047.20**	**99.00**
4223	162 743.55	99.70	5997	141 827.01	99.30	7015	127 501.12	98.90
9156	161 444.91	99.70	3813	141 695.26	99.30	2905	127 367.27	98.90
8912	160 969.21	99.70	6220	141 480.37	99.30	5296	127 154.49	98.90
3081	159 132.75	99.70	4188	141 034.87	99.30	7309	127 027.85	98.90
6459	158 472.18	99.70	2130	140 567.93	99.30			
9335	158 312.15	99.70	473	138 815.48	99.30			
2750	158 260.47	99.60	7184	138 582.06	99.30			
7056	157 442.74	99.60	9784	138 560.63	99.30			
28	156 828.69	99.60	3141	138 145.40	99.20			
2897	156 277.99	99.60	159	138 132.77	99.20			
1625	156 195.92	99.60	8990	137 726.73	99.20			
8	155 410.51	99.60	9558	137 318.94	99.20			
733	155 076.53	99.60	211	136 903.18	99.20			
4191	154 284.97	99.60	8981	136 337.58	99.20			

Source: prepared by the authors.

Graph 13
Adjusted Poisson



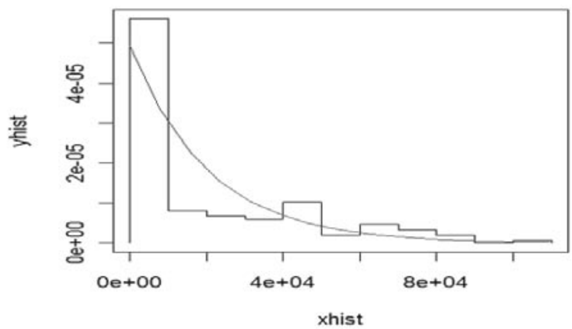
Source: prepared by the authors.

The χ^2 test had the following results: $\chi^2 = 10.86$, $g - 1 = 3$ and $p = 0.0043$. The χ^2 value in tables for a 99% confidence level and 3 degrees of liberty is 11.34, implying acceptance of the null hypothesis that posits that the sample comes from a Poisson with a $\lambda = 0.6$ parameter.

Traditional severity analysis

The amount of daily losses due to operational risk events in the securities settlement process is adjusted with an exponential distribution with an r= 4.930874e-05 parameter, as shown by the following graph.

Graph 14
Adjusted exponential



Source: prepared by the authors.

We carried out the χ^2 text with the following results: $\chi^2 = 4.6377$, $g - 1 = 5$ y $p = 0.4617$. The p-value is greater than 5% and so we conclude that the sample comes from an exponential with a r = 4.930894e-05 parameter.

Calculating the operational-risk value with the traditional model

By means of a Monte Carlo simulation, we "integrate" the traditional frequency and severity distributions so as to generate a potential-loss distribution (using a Poisson with a $\lambda = 0.6$ parameter for frequency and an exponential with a $1/r = 20\,280$ average for severity) in the stock market settlement process.[11] For the OpVar calculation, the values obtained for expected losses are arranged in descending order and the corresponding percentiles calculated. Results with confidence levels above 98.9% are shown below.

If the OpVaR is calculated with a confidence level of 99%, the maximum daily expected loss is 100 511 pesos with the traditional model. Thus the OpVaR calculation for the operational loss of the Mexican stock market's settlement process is 223 767 pesos per day. These results show that the OpVaR calculated with the Bayesian model is greater than that calculated with the traditional model, which can be explained by the causality between the various risk factors, which is not taken into account in the traditional model.

Conclusions

Currently financial institutions generate large amount of information arising from their interactions with clients, and from within the financial sector itself and its internal processes. Yet insufficient attention had been given to the dealings of individuals with the processes and to information systems. This concern was expressed by the Bank for International Settlements and addressed by Basel II, which asked that relevant institutions establish firm methodologies for measuring and administering operational risk.

Within this framework, this paper offers the necessary theoretical grounding and a practical guide to identify, measure, quantify, and administer the OR in the financial sector by using a Bayesian approach. In the course of this paper, this approach showed that it incorporates elements more closely linked to reality, such as: probabilities provided by experts when no historical information exists; specific probability distributions for each risk factor, be they discrete or continuous; updating of data incorporated into the model, and the interrelation

[11] Readers interested in reviewing the simulation results are asked to contact the authors by email.

Table **19**
Percentiles for the traditional model

Position	*Loss*	*Percentage*	*Position*	*Loss*	*Percentage*	*Position*	*Loss*	*Percentage*
1217	199 745.04	100.00	5752	128 825.30	99.60	9992	106 439.50	99.20
9678	184 375.55	99.90	5988	127 719.47	99.60	6948	106 283.43	99.20
2654	178 109.30	99.90	9784	127 018.89	99.60	8653	106 225.21	99.20
2433	172 579.05	99.90	2897	126 183.74	99.60	7376	106 100.30	99.20
2398	170 223.61	99.90	9394	124 109.63	99.50	8329	105 865.60	99.20
9923	169 487.34	99.90	1041	123 560.28	99.50	6455	105 135.34	99.20
9474	168 414.57	99.90	4534	123 487.61	99.50	5472	104 787.33	99.20
8964	155 980.20	99.90	8370	121 312.84	99.50	3292	104 413.41	99.20
4032	154 919.70	99.90	5967	120 423.85	99.50	5851	104 136.13	99.10
3975	151 986.89	99.90	5217	120 308.48	99.50	5642	103 735.02	99.10
2613	151 062.46	99.80	7355	119 451.59	99.50	6822	103 686.15	99.10
9666	146 741.97	99.80	9080	119 184.14	99.50	513	103 369.79	99.10
3877	146 052.97	99.80	6175	118 659.98	99.50	7904	103 361.83	99.10
5556	145 415.17	99.80	1500	116 322.99	99.50	7870	103 072.23	99.10
722	145 251.10	99.80	8633	115 948.08	99.40	3068	103 037.19	99.10
4738	143 112.37	99.80	7482	114 098.06	99.40	3763	103 026.49	99.10
6918	140 978.45	99.80	6819	114 073.09	99.40	7933	102 752.43	99.10
3715	139 229.28	99.80	3612	113 114.73	99.40	4359	102 163.58	99.10
1170	137 400.07	99.80	1439	112 991.05	99.40	8954	102 017.97	99.00
8176	135 567.61	99.80	1318	112 838.07	99.40	2246	101 835.59	99.00
5396	135 256.35	99.70	9751	112 750.33	99.40	2058	101 787.87	99.00
9170	135 230.78	99.70	2428	112 547.74	99.40	1632	101 482.69	99.00
3116	135 118.79	99.70	7430	112 048.58	99.40	3282	101 438.75	99.00
1241	135 052.84	99.70	9646	111 800.92	99.40	3203	101 185.93	99.00
4418	134 661.51	99.70	617	111 585.86	99.30	5961	101 083.72	99.00
6165	133 765.73	99.70	5301	110 048.20	99.30	2101	100 940.97	99.00
5955	133 216.33	99.70	9351	109 885.68	99.30	5747	100 911.96	99.00
5177	132 448.04	99.70	8112	109 718.99	99.30	**7943**	**100 511.14**	**99.00**
1218	132 053.30	99.70	859	109 464.93	99.30	724	100 456.09	98.90
6205	131 942.38	99.70	1828	109 163.21	99.30	5316	100 271.60	98.90
7247	130 965.73	99.60	9406	109 100.90	99.30	7754	100 076.60	98.90
3328	130 518.02	99.60	9360	108 234.32	99.30	9208	99 598.29	98.90
3799	130 334.15	99.60	9550	108 128.27	99.30			
8472	129 544.93	99.60	4325	107 944.03	99.30			
9510	129 210.27	99.60	6201	107 621.53	99.20			
8103	129 111.86	99.60	2080	107 401.71	99.20			

Source: prepared by the authors.

(causality) of the risk factors, by means of network models. We succeeded in demonstrating that Bayesian networks are a viable option for administrating operational risk in a climate of uncertainty and of scarce information or of doubtful quality. Yet the information obtained from experts can generate bias or inconsistency, for which reason it is vital to have solid and reliable educational tools,[12] among them the search conference, an analysis of business processes, and the Delphi technique.

Capital requirements for operational risk, as calculated for INDEVAL, are based on the assumption of an interrelation between risk factors (cause-effect), which is consistent with reality. For example, when we analyzed the failure-criticality node, we saw that the event is dependent on the back-office-system-failure and back-office-system-inactivity nodes. Calculating the operational value at risk by means of the traditional statistical method does not consider the interrelation or causality among the diverse risk factors, thus sub-estimating the maximum expected loss in terms of the required capital in the Bayesian model. In a scenario of extreme losses, the results of the traditional model would affect the operating viability of the securities settlement procedure.

By construction, a Bayesian network includes market information to calibrate the model. In addition, the BN is dynamic and needs to have up-to-date and reliable information, thus requiring a knowledge base from which the model can draw on systematically.

The maximum expected loss due to operational risk calculated for INDEVAL's settlement process is a relatively low amount, compare to the daily volume of transactions. Yet it reflects the high service and security standards with which the securities settlement procedure operates, and is coherent with the systemic transcendence of one of the most important settlement procedures in Mexico.

Bayesian networks are based on efficient evidence diffusion algorithms that dynamically update the model with real data. For this paper's purpose, it was possible to build a BN and calculate the required capital to administer the operational risk, combining statistical data and INDEVAL experts' opinions or judgments.

Within this study, the number of nodes that make up the networks requires few probability calculations and, thus, the junction tree algorithms used herein

[12] By educational techniques we refer to heuristic techniques to obtain quality information from experts that will allow us to determine subjective probabilities or beliefs regarding the probability of some event occurring.

is the most appropriate. To analyze problems of greater complexity, more processing power is required and we would thus recommend approximate-solution algorithms, such as the Markov Chain Monte Carlo (MCMC).

The conditional OpVaR calculated with the Bayesian approach is consistent in the sense of Artzner, but it also summarizes the complex causal relationships among the different risk factors that arise from an operational risk event. In summary, since reality is much more complex than identically-distributed independent events, the Bayesian approach has advantages over the traditional manner of modeling a complex and dynamic reality.

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