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Single and multi-objective optimization of path placement for redundant robotic manipulators

Optimización mono-objetivo y multi-objetivo del emplazamiento de trayectorias de robots manipuladores redundantes

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Abstract

General formulations are presented in this paper to determine the best position and orientation of a desired path to be followed by a redundant manipulator. Two classes of problem are considered. In the first, a single manipulator's index of kinematic performance associated to one path point must be improved as much as possible. In the second case distinct indices of kinematic performance, corresponding to different points of the path, are to be optimized. Constraints are taken into account in order to guarantee the accessibility to the whole desired task. Several case studies are presented to illustrate the effectiveness of the method for planar and spatial manipulators.

Keywords: *Optimization, redundant manipulators, path placement, motion planning, kinematic performances.*

Resumen

En este artículo se presentan formulaciones generales para determinar la mejor posición y orientación de una ruta que se desea que recorra el órgano terminal de un manipulador redundante. Se consideran dos clases de problemas. En el primer caso un índice de desempeño cinemático, asociado a un solo punto de la trayectoria, debe mejorarse tanto como sea posible. En el segundo caso se optimizan distintos índices de desempeño cinemático, correspondientes a diferentes puntos de la ruta deseada. Se consideran restricciones para garantizar la accesibilidad a toda la ruta deseada. Para ilustrar la efectividad del método se presentan varios casos de estudio de manipuladores planares y espaciales.

Descriptores: Optimización, manipuladores redundantes, colocación de trayectorias, planificación de movimientos, desempeño cinemático.

I. Introduction

A robot manipulator is kinematically redundant if it has more degree-of-freedom (dof) than the minimum required for the accomplishment of its tasks. The redundancy increases the ability of the robot to avoid collisions as well as singular or degenerated configurations when a task is carried out. However, this class of manipulators requires more complex schemes for motion planning compared with non

redundant manipulators. Indeed, a redundant manipulator should move in such a way that the end-effector achieves a desired main task and the rest of the arm simultaneously accomplishes a secondary task. The secondary task may be defined as internal motions to optimize manipulator's performances, or to avoid collisions. The kinematic performances can be measured in terms of criteria chosen by the user, like the manipulability (Yoshikawa , 1985) or the condition number (Angeles *et al*., 1992) of the Jacobian matrix, which are interesting in certain applications.

The redundancy of manipulators has been solved in the literature by optimizing locally (Yoshikawa *et al*., 1989), (Chiu, 1988) or globally (Nenchev, 1996), (Nakamura *et al*., 1987), (Pamanes *et al*., 1999) the kinematic or dynamic performances. In such works it is assumed that the path placement is specified with respect to the robot's frame. Therefore, the performances of the manipulator obtained by applying these methods are relative to the assumed path location. Nevertheless, in some applications, the user could find a suitable path location to improve as much as possible the robot's performances. An automatic turning table or a collaborative manipulator can be used as positioner devices in the robotic workstation to judiciously place the task to the main robot.

The subject of the optimal relative robot/path placement has been studied in the literature mainly for non-redundant manipulators (Zhou *et al*., 1997), (Nelson *et al*., 1987), (Fardanesh *et al*., 1988), (Pamanes, 1989), (Pamanes *et al*., 1991), (Reynier *et al*., 1992), (Abdel-Malek, 2000). To the author's knowledge, only *J.S. Hemmerle* and *F.B. Prinz* (Hemerle *et al*., 1991) considered the problem of the optimal path placement for redundant manipulators; in this study, it is assumed that the task is held by a collaborative manipulator (*left hand*) moving simultaneously with the main manipulator (*right hand*) which drives the tool. Two criteria of optimization were simultaneously considered: the joint variables were moved away from their limit values as much as possible and the joint displacements were minimized. In that method, however, constraints were not taken into account to assure continuous joint trajectories. On the other hand, the case was not studied in which the task doesn't move simultaneously with the main manipulator; besides, the optimization of kinematic performances on specific zones of the path was neither investigated. The resolution of both problems becomes interesting in industrial applications.

Two cases of optimal path placement are studied in this paper for redundant manipulators. In the former (*single-objective problem*) we formulate a process to compute the path placement which allows to optimize one index of kinematic performance of the manipulator on only one point of the desired path. In the second case (*multi-objective problem*) we compute the path placement such that distinct indices of kinematic performance are optimized on different zones of the path. Constraints are taken into account in order to avoid both exceed the joint limits and discontinuous joint trajectories.

In the next section some concepts of robot kinematics are evoked which are later applied in our formulations. A solution is presented in the third section for the single-objective problem and then, in the fourth part of the paper, the multi-objective problem is studied. The generalization of our formulations to solve the case of global optimization is observed in the fifth section. Some case studies are discussed to illustrate the effectiveness of the methods for both planar and spatial manipulators. The concluding remarks are finally presented.

II. Preliminaries

The kinematic function of a robot manipulator is defined as:

$$\boldsymbol{x} \quad \boldsymbol{f(q)} \tag{1}$$

where $\boldsymbol{x}$ is the m-dimensional vector of operational coordinates describing the situation of the end-effector in the Cartesian space; $\boldsymbol{q}$ is the n-dimensional vector of joint variables defining the instantaneous configuration of the manipulator; $\boldsymbol{f}$ is an m-dimensional function (n m).

The inverse kinematic function of a manipulator, if it exists, is given by

$$\boldsymbol{q} \quad \boldsymbol{f}^{-1}(\boldsymbol{x}) \tag{2}$$

The direct velocity function of a robot manipulator is obtained by differentiation of equation 1:

$$\dot{\boldsymbol{x}} = \boldsymbol{J}(\boldsymbol{q})\dot{\boldsymbol{q}} \tag{3}$$

where the dot denotes differentiation with respect to time and $\boldsymbol{J}(\boldsymbol{q})$ $\boldsymbol{R}^{m\ n}$ is the Jacobian matrix of the manipulator. When the number n of joint variables q_i of a manipulator is equal to the number m of operational coordinates x_j of the end effector, then the manipulator is called *non redundant.* On the other hand, when $n > m$ the manipulator is termed redundant. In this case the inverse kinematic function of equation (2) has an infinite number of solutions.

The inverse velocity function of a manipulator is obtained from equation 3 as

$$\dot{\boldsymbol{q}} \quad \boldsymbol{J}^{1}(\boldsymbol{q})\dot{\boldsymbol{x}} \tag{4}$$

If $\boldsymbol{J}(\boldsymbol{q})$ is singular or $n > m$ then the inverse $\boldsymbol{J}^{-1}(\boldsymbol{q})$ does not exists and the linear system of equation (3) cannot be solved by using equation (4). In such a case the inverse velocity function may be expressed as

$$\dot{\boldsymbol{q}} \quad \boldsymbol{J}\ \dot{\boldsymbol{x}} \quad (\boldsymbol{I} \quad \boldsymbol{J}\ \boldsymbol{J})z \tag{5}$$

where

$\boldsymbol{J}$ is the pseudo-inverse of $\boldsymbol{J}$ (*in order to simplify the terms in this paper* $\boldsymbol{J}(\boldsymbol{q})$ *will be equivalent to* $\boldsymbol{J}$).

$\boldsymbol{z}$ *is an arbitrary vector* $\boldsymbol{R}^{n}$.

$\boldsymbol{I}$ is the identity matrix of dimension n´n.

In equation (5), $\boldsymbol{J}\ \dot{x}$ is the *least-norm solution* of equation (3), i.e., it provides a vector with minimum Euclidean norm satisfying this Equation. On the other hand, $(\boldsymbol{I}-\boldsymbol{J}^{+}\ \boldsymbol{J})\ \boldsymbol{z}$ represents the projection of $\boldsymbol{z}$ on the null-space of $\boldsymbol{J}$; this part is called the *homogeneous solution* of equation (3); it is referred to as the self-motion of the manipulator and does not cause any end-effector motion. In order to use the self-motion to improve configurations, the vector $\boldsymbol{z}$ is chosen as

$$\boldsymbol{z} \quad k\ \ h(\boldsymbol{q}) \tag{6}$$

where

$h(\boldsymbol{q})$ is the gradient of an index of performance $h(\boldsymbol{q})$ to be optimized.

k is a scaling factor of $h(\boldsymbol{q})$. It is taken to be positive if $h(\boldsymbol{q})$ must be maximized and negative if $h(\boldsymbol{q})$ is to be minimized.

Several criteria of kinematic performances for manipulators have been proposed in the literature, which can be considered for the index $h(\boldsymbol{q})$ in equation (6). Each of such indices evaluates different kinematic features of a manipulator, which may be interesting depending on the nature of the task to be carried out. A succinct survey of two indices of performance is presented below. Such indices, the *manipulability* and the *condition number* of the jacobian matrix, will be applied as criteria of optimization to solve the path placement problems in section VI.

The *manipulability* index, introduced by Yoshikawa (1985), is defined as

$$w \quad \sqrt{\det(\boldsymbol{J}\boldsymbol{J}^{T})} \tag{7}$$

The manipulability is a measure of the ability to arbitrarily change the position or orientation of the end effector.

Thus, its maximization would be appreciated in task zones where relatively large deviations in the prescribed motion of the end-effector are likely.

The *condition number* of the Jacobian matrix is another interesting index applied to evaluate the performances of robotic manipulators (Angeles *et al*., 1992). Such index can be computed as:

$$C(\boldsymbol{J}) \quad \frac{\max}{\min} \tag{8}$$

where $_{max}$ is the largest singular value of $\boldsymbol{J}$ and $_{min}$ is the smallest singular value of $\boldsymbol{J}$.

The minimum condition number of a manipulator minimizes the error propagation from input joint velocities to output end-effector velocities. Thus, it can be used as a local measure of accuracy of the manipulator's motions.

III. Optimal path placement for single-objective optimization

A. Statement of the problem

The task of a n-dof manipulator is specified by a set of n_t m-dimensional vectors of operational coordinates of the end-effector in an orthonormal frame $_t$. The manipulator considered is assumed as being redundant ($n>m$). The n_t vectors correspond to a sample of points p_i (i = 1, 2, ..., n_t) of the desired path in the task space. The operational coordinates are the desired positions and orientations of the frame $_{n+1}$ attached to the end-effector, as showed in figure 1.

A suitable index of performance is then assigned by the user to one arbitrary path point, say p_k , k {1,..., n_t}, which must be maximized by the corresponding manipulator's configuration when the task will be accomplished. A law of motion is also given which refers to the time the position of the end effector on the path.

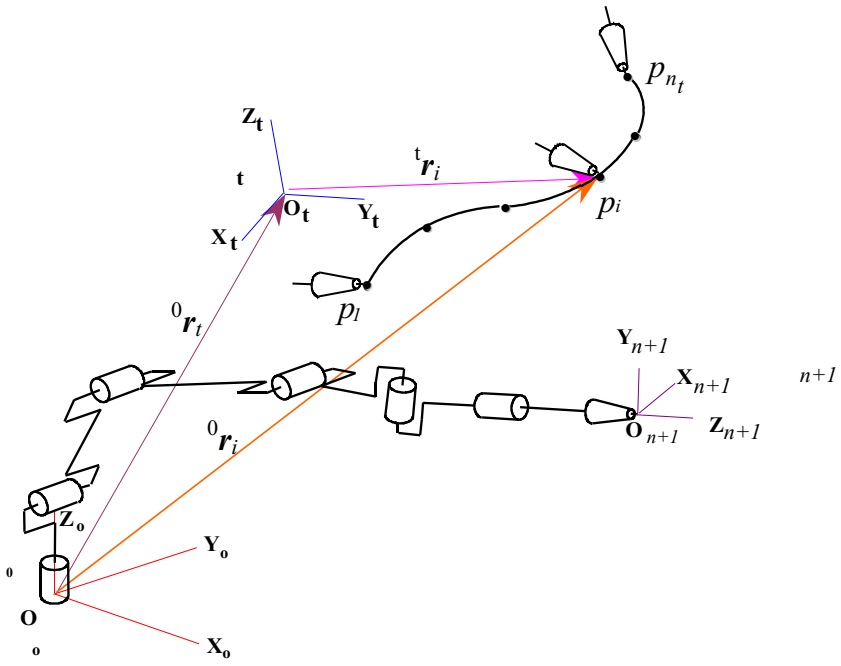

Figure 1. Desired path referred to the frame $_t$ and its placement in the frame $_0$ fixed to the robot

On the other hand, the path placement is specified by variables regarding the position and orientation of the frame $_t$ with respect to the frame $_0$ fixed to the base of the robot. They are the components of the *placement vector* defined as

$$^0e \quad r_{tx,}\, r_{ty}, r_{tz}, \;,\;,\; ^T \tag{9}$$

where r_{tx}, r_{ty}, r_{tz}, are the orthogonal components of the vector of position $^o\boldsymbol{r}_t$ (Figure 1), and $\;,\;,\;$ are the Euler angles Z-Y-X defining the orientation of $_t$ with respect to the frame $_0$.

It is desired to obtain the components of the placement vector $^0\boldsymbol{e}$ of the path, so that the index of manipulator's kinematic performance associated to the sample point p_k is optimized when the task is accomplished.

B. Process of solution

The single-objective problem is equivalent to a constrained non-linear programming problem. The objective function is defined as the index of performance $h_k(\boldsymbol{q}_k)$ assigned to the path point p_k. The number of independent variables will be generally $6+n$: the 6 components of the placement vector $^0\boldsymbol{e}$ of the path and, because of the manipulator's redundancy, the n joint variables of the configuration $\boldsymbol{q}_k$ which allow to satisfy the desired situation of the end-effector at the path point p_k.

To solve the problem, we propose an optimization process in three phases: *phase I* in which the optimal placement vector must be found; *phase II* addressed to compute the optimal configuration on the path point p_k for each proposed placement to be evaluated; and *phase III* committed to complete the manipulator's joint trajectories for the whole desired path by using the optimal path placement obtained in phase I. Notice that phase II allows to evaluate the objective function of phase I. Such a function in phase I can be defined as

$$f \quad h_k(\boldsymbol{q}_k) \tag{10}$$

The general procedure to solve the single-objective problem is schematized in the flow chart of figure 2. Details on the three phases concerned are given below.

Phase I

Before obtain the configurations at each path point, the operational coordinates must be referred to frame Ó$_0$. Therefore, when a new placement is proposed in the optimization process, the operational coordinates have to be first updated in phase I by applying the following transformation:

$$^{0}\boldsymbol{T}_i \quad ^{0}\boldsymbol{T}_t\,^{t}\boldsymbol{T}_i \tag{11}$$

In this equation:

$^{t}\boldsymbol{T}_i$ is the homogeneous matrix establishing the desired position and orientation of the end effector on the path point p_i referred to frame $_t$.

$^{0}\boldsymbol{T}_t$ is the homogeneous matrix for the position and orientation of frame $_t$ referred to frame $_0$.

$^{0}\boldsymbol{T}_i$ is the homogeneous matrix defining the position and orientation of the end effector on the path point p_i with respect to frame $_0$.

When the given operational coordinates have been updated, then the objective function must be evaluated in phase II for the current placement; after the process returns to phase I in order to check for the convergence of the method. If the convergence is attained then the procedure goes to phase III; otherwise, the placement must be improved by applying a suitable strategy.

Phase II

After updating of the operational coordinates for a path placement proposed in phase I, the redundancy must be solved to find the configurations $\boldsymbol{q}_i$ (i=1, 2, ... n_t) for all the path points. To do that, we assume as secondary task on point p_i the

optimization of the same index of performance considered on p_k; i.e., for one path point the manipulator has to achieve the corresponding operational coordinates and also optimize $h_k(\boldsymbol{q}_i)$. To find such configurations the following process is carried out:

1. Find a configuration $\boldsymbol{q}_i$ at each path point in such a way that Equation (1) is satisfied. This configuration is obtained by minimization of the following function:

$$e_{pos} \quad \left\| \boldsymbol{x}_i \quad \boldsymbol{x}_i' \right\| \qquad (12)$$

where $\boldsymbol{x}_i$ is the vector defining the desired situation of the end-effector at point p_i, and $\boldsymbol{x}_i'$ is a vector of operational coordinates corresponding to a configuration $\boldsymbol{q}_i'$ proposed when minimizing of equation (12). The symbol $\| \ \|$ denotes the Euclidean norm. If the task is feasible then equation $\boldsymbol{x}_i = \boldsymbol{x}_i'$ will be satisfied when the norm of equation (12) is minimized. The numeric minimization is carried out by applying a method of constrained non-linear optimization. The independent variables are the joint variables of $\boldsymbol{q}_i'$. The constraints to be considered are presented in the next sub-section.

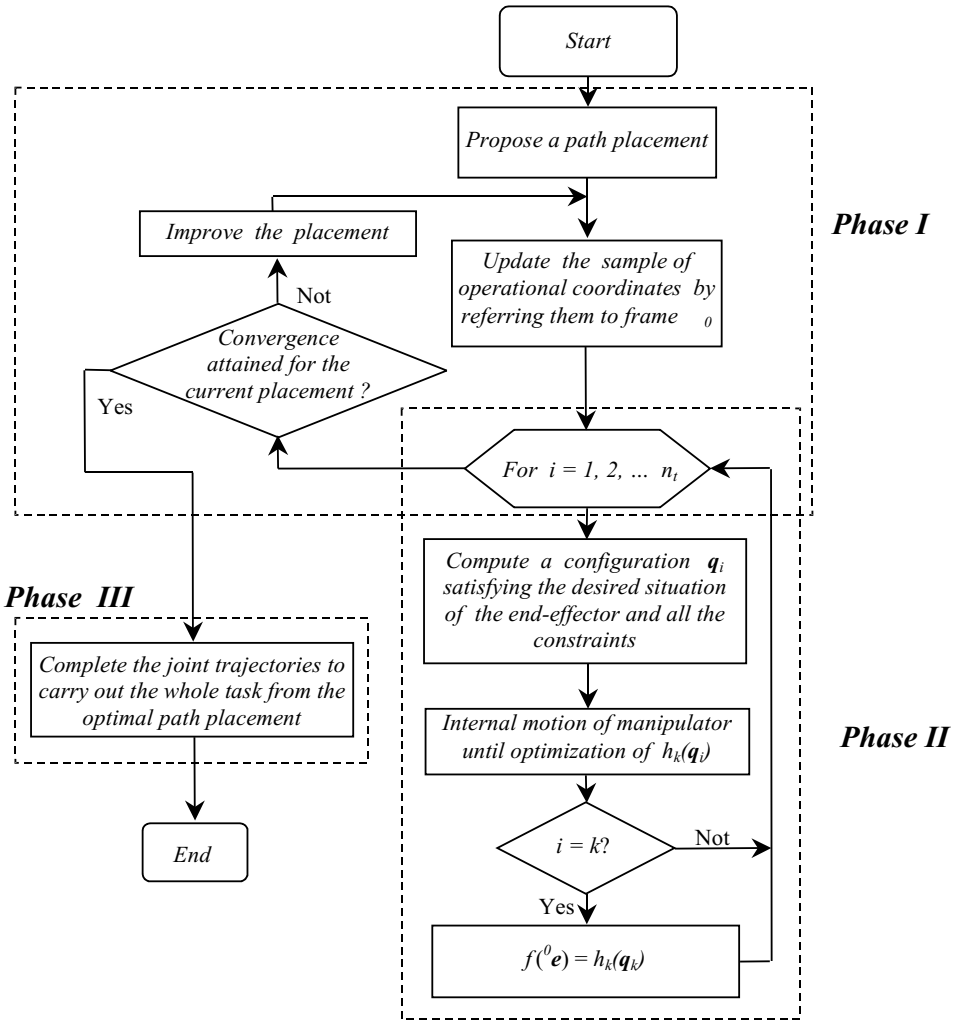

Figure 2. Computational algorithm for the single-criterion problem

2. When a configuration $\boldsymbol{q}_i$ at one path point has been found, satisfying both equation (1) and all the constraints, then compute $\boldsymbol{J}$, $\boldsymbol{J}$ and, $h(\boldsymbol{q})$ and optimize for such a configuration the index $h(\boldsymbol{q})$ by successively obtaining of the homogeneous solution of equation (3). At each iteration of this process, when a vector $\dot{\boldsymbol{q}}_i$ of the homogeneous solution is obtained corresponding to a certain configuration $\boldsymbol{q}_i$, an improved configuration $\boldsymbol{q}_i^*$ may be computed by

$$\boldsymbol{q}_i^* \quad \boldsymbol{q}_i \quad \dot{\boldsymbol{q}}_i \quad t \tag{13}$$

where is a sufficiently small time interval. Note that, because $\dot{\boldsymbol{q}}_i$ is a homogeneous solution, the improved configuration $\boldsymbol{q}_i^*$ will also preserve equation (1). The initial configuration of the optimization process has been determined in step **i.**

3. For one path-point the optimization of a configuration is stopped when $\| h(\boldsymbol{q}) \|$ becomes zero.

Phase III

When computing the optimal path placement only a significant sample of path points is considered in order to reduce the time of computation engaged in the optimization process. Nevertheless, the desired trajectory is a continuous curve which must be approximated by a sufficiently large number of supplementary points of the path. So, to synthesize continuous joint trajectories for the whole task, when the optimal placement has been found the redundancy must be solved for supplementary path points. This process is the same used to solve the redundancy in phase II by optimizing the desired index of performance $h_k(\boldsymbol{q})$. The number of supplementary points is proposed by the user in such a way that a conventional precision be satisfied. Continuous joint trajectories will be obtained as a result of this process because the index to be optimized is the same for all the considered points.

C. Constraints of the problem

The optimization processes to obtain the path placement and to solve the redundancy must be constrained in order to obtain realistic solutions. The following constraints are taken into account:

1. Explicit constraints in phase I

Explicit constraints are imposed in phase I on the placement vector so that solutions are obtained only into the available physical space for the task placement. Such space may be imposed by a positioner device. The following constraints on the components of the placement vector are considered:

$$r^0_{t\,x(l)} \quad r^0_{t\,x} \quad r^0_{t\,x(u)} \tag{14a}$$

$$r^0_{t\,y(l)} \quad r^0_{t\,y} \quad r^0_{t\,y(u)} \tag{14b}$$

$$r^0_{t\,z(l)} \quad r^0_{t\,z} \quad r^0_{t\,z(u)} \tag{14c}$$

$$(l) \qquad (u) \tag{14d}$$

$$(l) \qquad (u) \tag{14e}$$

$$(l) \qquad (u) \tag{14f}$$

In expressions (14), the indexes l and *u* denote, respectively, lower and upper bounds of the independent variables.

2. Implicit constraints in phase I for access to the task

Implicit constraints are also considered in order to guarantee the efficacy of placements proposed in the optimization process. To assure the accessibility to all the sample points on the path the following constraint is introduced:

$$\|\boldsymbol{t}_i\| \quad t_u \quad i \quad 1,2,\ldots,n_t \tag{15}$$

where $\boldsymbol{t}_i$ is the *reach* vector demanded to the manipulator at point p_i; the symbol $\| \ \|$ denotes Euclidean norm. t_u is the norm of the maximum reach which the manipulator can provide. Such norm depends on the geometric parameters of the manipulator.

3. Explicit constraints in phase II for joint limits avoidance

An elemental condition for any feasible configuration consists in preserving the joint variables into the admissible domain of configurations. So, any configuration used for the task should verify the following conditions:

$$q_i^{(l)} \quad q_{ij} \quad q_i^{(u)} \quad i \quad 1,\ldots,n; \quad j \quad 1,\ldots,n_t \quad (16)$$

where:

q_{ij} is the *i*-th joint variable of the $\boldsymbol{q}_j$ manipulator's configuration corresponding to the *j*-th task point.

$q_i^{(l)}\, q_i^{(u)}$ are the lower and upper limits, respectively, of the *i*-th joint variable.

4. Implicit constraints in phase II to guarantee continuous joint trajectories

Implicit constraints are imposed in phase II which allow to assure the feasibility of the whole joint trajectories.

To introduce the considered constraints we recall here de notion of the *aspect* of a manipulator. One aspect is defined (Borrel *et al*., 1986) as a continuous subset of the manipulator's joint space composed by configurations which render non-zero all the *m*-order minors of the jacobian matrix, except those minors being zero everywhere in the joint space.

Thus, the aspects of a manipulator are subsets of the joint space separated by hypersurfaces whose equations are determined by the *m*-order minors of the jacobian matrix equalized to zero.

On the other hand, it is known that for *non-cuspidal* manipulators (Burdick *et al*., 1995), (Wenger, 1997), the continuity of joint trajectories corresponding to a given task can be guaranteed if the manipulator is constrained to use configurations remaining in only one aspect of its joint space.

Consequently, to guarantee continuous joint trajectories the following conditions must be imposed to manipulator's configurations which will be used to accomplish a desired path:

$$_{kj} \quad _{kj}(\boldsymbol{q}) \quad 0 \quad (17)$$

where

$_{kj}(\boldsymbol{q})$ is the left hand function of the equation defining the j-th hypersurface ($j = 1, 2,..., n_{hk}$) which borders the aspect A_k in the joint space; $k \quad \{1, 2, \ldots, n_A\}$. n_{hk} is the number of such hypersurfaces. n_A is the number of the robot's aspects. Only one aspect will be chosen containing all the configurations which allow to achieve the desired task.

$_{kj}$ is a constant (+1 or –1) depending on the hypersurface and the considered aspect A_k.

In section VI we will identify the implicit constraints (17) for two manipulators.

VI. Optimal path placement for multi-objective optimization

A. Statement of the problem

The task of a *n*-dof manipulator is specified by a set of n_t *m*-dimensional vectors of operational coordinates of the end-effector ($n>m$) in an orthonormal frame $_t$. Such operational coordinates give the desired positions and orientations to be followed by a frame $_{n\,1}$ attached to the end-effector, as showed in figure 1. In that figure a sample of points p_i (i = 1, 2, ..., n_t) is illustrated corresponding to the desired path in the task space. Suitable indexes of performance are then assigned by the user to several path points p_i. Thus, we want to compute the path placement vector $^0\boldsymbol{e}$, so that all the indexes associated to the sample points be optimized while the task is being accomplished. A law of motion is also specified which refers to the time and position of the end-effector on the path.

B. Process of solution

The multi-objective problem is also a constrained non-linear programming problem. The objective function should consistently characterize a set of dissimilar indexes of performance to be optimized. Thus, it is required that the indexes be first normalized to eliminate scaling and unity differences; then they can be included in a coherent objective function.

As in the single-objective problem, the independent variables will be the 6 components of the path placement vector ${}^0\mathbf{e}$ and, because of the manipulator's redundancy, the joint variables of configurations $\boldsymbol{q}_i$ which allow to satisfy the desired situation of the end-effector on the path points p_i. To solve the multi-objective problem we propose an optimization process having also three phases. In *phase I* the optimal placement will be searched; *phase II* is addressed to compute the optimal configurations at the n_t path points for each placement to be evaluated; *phase III* is finally committed to synthesize the manipulator's joint trajectories for the whole desired path by using the optimal path placement obtained in phase I. Note that phase II allows to evaluate the objective function of phase I. Such a function is defined below. The procedure presented here to solve the multiobjective problem is illustrated in the flow chart of Figure 3. Details of the procedure are discussed in the next paragraphs.

Phase I

The path placement must be searched in this phase. For each placement proposed here we have to update the operational coordinates; then an evaluation of the placement can be accomplished in the process of optimization.

The transformation of such coordinates is carried out by applying equation (11); then the redundancy will be solved in phase II, and the objective function will be computed based on normalized indexes of performance. After that the process returns to Phase I in order to check for the convergence of the method. If the convergence is attained then the procedure goes to Phase III; otherwise, the placement must be improved by applying a suitable strategy. The objective function for the multi-objective problem as well as the normalized indexes, are defined below.

A normalized index of performance associated to the sample point p_i is obtained as:

$$c_i \quad \frac{h_i(\boldsymbol{q}_i)}{h^*_i} \qquad i \quad \{1,2,\ldots,n_t\} \tag{18}$$

The normalization factor h_i^* in equation (18) is the maximum value of the index of performance at the point p_i that can be obtained by the manipulator when only such index is optimized, and the accessibility to all the path points is satisfied. In fact, the normalization factor h_i^* is the optimal value obtained for the index $h_i(\boldsymbol{q}_i)$ in the single-objective problem. Thus, to obtain the normalization factors we have to solve as much single-objective problems as sample-configurations are to be optimized. It must be observed that a sample-point could hold or not an index of performance associated to be optimized; thus, the number of indexes to be optimized can be lower or equal than n_t.

The main idea to define the objective function is that the value of such function, corresponding to a path location, must be equivalent to a typical value of the set of normalized indices to be optimized. Such typical value can be defined as:

$$c \quad \bar{c} \quad \hat{}_c \tag{19}$$

where $\bar{c}$ and $\hat{}_c$ are, respectively, the mean and the standard deviation of the set of normalized indexes associated to the sample points. Hence, the value of c obtained by equation (19) corresponds to a typically small value of the set of normalized indexes of the sample.

If we assume that the all the indexes of performance considered in the problem are to be

maximized, then the global maximization of the set of n_t indexes would be obtained by maximization of c. Nevertheless, algorithms in usual software for optimization are generally intended to minimize the objective function. Thus, to solve the optimization problem by minimization, the objective function could be defined as $-c$; therefore, such a function can be finally written as:

$$f \quad \hat{}_c \quad \bar{c} \tag{20}$$

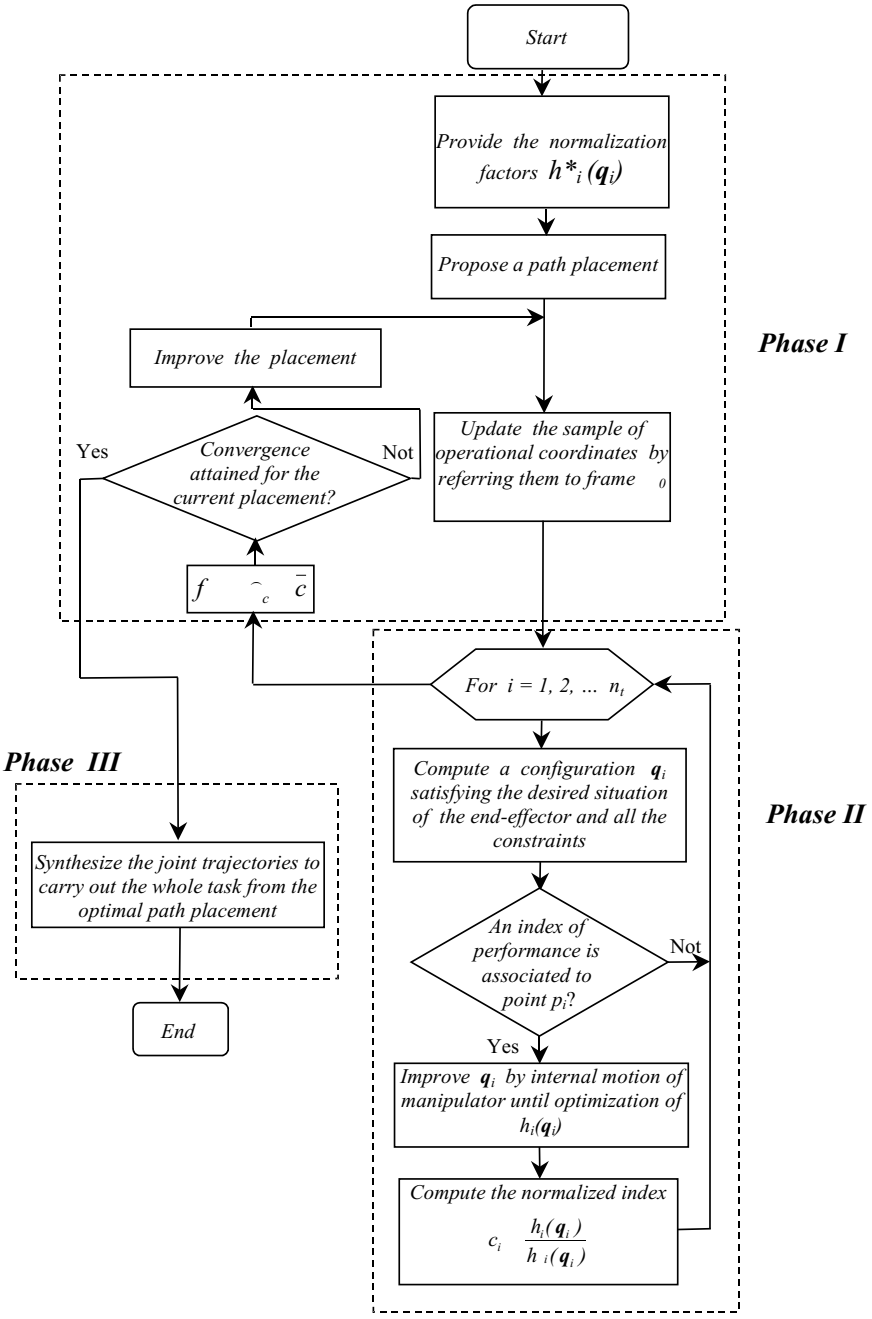

Figure 3. Algorithm for multi-criteria problem

Phase II

The n_t optimal configurations satisfying the desired task and constraints, which are used to compute the normalized indexes, must be found in phase II. The algorithm to compute such configurations is the same used in single-objective problem; this one has been described in section III.

Phase III

The continuous joint trajectories for the whole task must be finally synthesized in Phase III for the optimal path placement. To do that, the redundancy has to be solved for supplementary path points in such a way that the optimal configurations of points p_i obtained in phase II are preserved.

In the multi-objective problem however, because of generally different indexes of performance are associated to adjacent sample path points, say p_i and p_{i+1}, we cannot use a single index to solve the redundancy by using the homogeneous solution for intermediate path points. In fact, if the homogeneous solution is applied on intermediate points to optimize the index associated to p_i, then the joint trajectories becomes discontinuous on p_{i+1} and vice versa.

To solve the redundancy and synthesize the continuous joint trajectories connecting adjacent sample path points, a judicious strategy must be used. We propose a suitable secondary task to be accomplished, which is specified in the joint space as a continuous trajectory between configurations asso- ciated to adjacent sample path points. This secondary trajectory is such that the determinant of $\boldsymbol{J}\boldsymbol{J}^T$ smoothly evolves from its value on point p_i to the value on point p_{i+1}. Consequently, because all the configurations so obtained belong to only one aspect, the continuity of joint trajectories will be assured. Thus, to accomplish both the main and the secondary tasks, we propose to solve the redundancy by minimizing the following objective function at each intermediate point p_j between two successive sample points p_i and p_{i+1}:

$$f^*_j \quad e_{pos\,j} \quad e_{\det \boldsymbol{J}^*_J}, \quad j \quad 1,2,\ldots,n_{\text{int}} \tag{21}$$

where n_{int} is the number of intermediate points to be considered between two sample points. This number is chosen by the user so that a conventional precision be satisfied. The error of position of the end-effector, $e_{pos\,j}$, in equation (21), is defined like in equation (12) for intermediate points. On the other hand, we define $\boldsymbol{J}^*_j \quad \boldsymbol{J}_j\boldsymbol{J}_j^T$, where $\boldsymbol{J}_j$ is the Jacobian matrix of the manipulator on the intermediate point p_j. Then the error of the determinant of $\boldsymbol{J}^*_j$ in equation (21) is defined as

$$e_{\det \boldsymbol{J}^*_j} \quad abs \;\; det\, \boldsymbol{J}^*_J \quad det(\boldsymbol{J}^*_J)' \tag{22}$$

where:

$det\, \boldsymbol{J}^*_J$ is the desired value of the determinant of $\boldsymbol{J}^*$ for the configuration at point p_j. Its value is assigned by a cycloidal law [Equation (23)] which is defined for the current segment of the path between two successive sample points. Such cycloidal law must smoothly change the determinant of $\boldsymbol{J}^*$ from its value corresponding to the configuration at point p_i, to that one at p_{i+1} .

$det\,(\boldsymbol{J}^{*'}_J)$ is the value of the determinant of $\boldsymbol{J}^*$ for the current configuration in minimization of equation 21.

The desired behavior of the determinant of $\boldsymbol{J}^*$ in the segment between sample points p_i and p_{i+1} is described by the following cycloidal law:

$$det\, \boldsymbol{J}^*_J \quad det\, \boldsymbol{J}^*_i \qquad (23)$$

$$(det\, \boldsymbol{J}^*_{'i} \;\; \frac{t'_{pj}}{T_i} \quad \frac{1}{2} sen \;\; \frac{2\;t'_{pj}}{T_i}$$

The variables concerned in equation (23) are defined as follows:

$$(det\, \boldsymbol{J}^*_{'j} \quad det\, \boldsymbol{J}^*_{i\;1} \quad det\, \boldsymbol{J}^*_i$$

$t'_{pj} \quad t_{pj} \quad t_{pi}$

$T_i \quad t_{pi\ 1} \quad t_{pi}$

where t_{pi}, t_{pj} are the values of the time elapsed when the end-effector arrives at points p_i (sample) and p_j (intermediate), respectively.

C. Constraints of the problem

As considered for the single-objective problem, to obtain realistic solutions in solving the multi-objective problem, explicit and implicit constraints should also be taken into account. Such constraints are identical to those considered in Section III C. They were examined in that section.

V. Path placement for global optimization

The path placement problem was studied in section III for the optimization of manipulator's performance on a certain point of the path by taking into account a single kinematic criterion. Nevertheless, in some tasks the optimization of such criterion can be preferred not only on a particular point but on every one of points in the path; i.e. a *global optimization* of manipulator's performances is desired when the task is carried out. Note that such problem can be considered as a particular case of the multi-objective problem examined in the previous section. In fact, in the formulation for multi-objective optimization we can assign the same criterion of performance to all the sample points in order to attain the global optimization. However, it must be pointed out that the homogenization of the corresponding indices is not required in the process of solution; then equation (18) takes the form $c_i \quad h_i(\boldsymbol{q}_i)$.

Furthermore, after optimizing the set of indexes on the sample points, the continuous trajectories of joint variables for the whole path can be synthesized by applying only the Phase III of the process for the single-objective problem. Consequently, the minimization of function (21) is not necessary for global optimization.

VI. Case studies

Several case studies are presented in this section in which single and multi-objective problems of optimal path placement are solved; we consider planar and spatial paths for each kind of problem. The planar task must be accomplished by a 3R manipulator, and the spatial task should be achieved by a 4R manipulator. In the following sub-section, the geometric parameters of both manipulators will be specified and the implicit constraints of the problems to hold configurations in an aspect will be identified. Then, in succeeding subsections the problems will be solved.

A. Manipulators for the case studies

1. The 3R manipulator

The considered planar manipulator is shown in Figure 4, and its geometric parameters are presented in table 1. These parameters are defined by using the modified Denavit-Hartenberg method (Khalil *et al.*, 1986). A frame $_4$ is attached at the tip of the third link in order to use its origin to specify the linear velocity of the end-effector. The manipulator's joint variables are $\grave{e}_1$, $\grave{e}_2$ and $\grave{e}_3$, and its limit values are $^{(l)}_i$ 150 and $^{(u)}_i$ 150 , for $i = 1, 2, 3$.

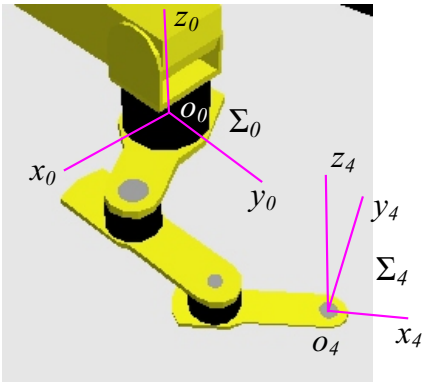

Figure 4. 3R Manipulator to be applied for planar tasks

Table 1. Geometric parameters of the 3R manipulator

		d		r
1	0	0	$_1$	0
2	0	d	$_2$	r_2
3	0	d	$_3$	r_3
4	0	d	0	0

By considering the velocity vector of O_4 referred to the frame $_0$ as $\dot{\boldsymbol{x}}$ $[\dot{x},\dot{y}]^T$, the jacobian matrix of the manipulator is:

$$\boldsymbol{J} \quad \begin{matrix} (s_1 \quad s_{12} \quad s_{123})d| \quad (s_{12} \quad s_{123})d| \quad s_{123}d \\ (c_1 \quad c_{12} \quad c_{123})d|(c_{12} \quad c_{123})d|c_{123} \quad d \end{matrix} \tag{24}$$

In this matrix and hereafter we use the following compact notation:

c_{ijk} cos($_i$ $_j$ $_k$) s_{ijk} sin($_i$ $_j$ $_k$)

c_{ij} cos($_i$ $_j$) s_{ij} sin($_i$ $_j$)

c_i cos($_i$) s_i sin($_i$)

The 2-order minors of the jacobian matrix are expressed as:

$$m_1 \quad d^2(s_2 \quad s_{23}) \tag{25a}$$

$$m_2 \quad d^2(s_3 \quad s_{23}) \tag{25b}$$

$$m_3 \quad d^2 s_3 \tag{25c}$$

The conditions of configurations which render zero the above minors provide the following equations of the surfaces dividing the joint space in aspects:

$$s_2 \quad s_{23} \quad 0 \tag{26}$$

$$s_3 \quad s_{23} \quad 0 \tag{27}$$

$$s_3 \quad 0 \tag{28}$$

Thus, six aspects can be identified (n_A=6) for the 3R manipulator, as showed in figure 5. We chose the aspect A_1 for the achievement of the task; consequently for (17) we have k=1. Then we observe that the aspect A_1 is surrounded by the surfaces specified by equations (26) and (28); therefore we have n_{hk} =2 and $\mathring{a}_{11}$=1, $\mathring{a}_{12}$=1 for ine- quality (17).

The implicit constraints on $\grave{e}_2$ and $\grave{e}_3$ to hold configurations in the aspect A_1 are then expressed as

$$_{11} \quad 0 \tag{29}$$

and

$$_{12} \quad 0 \tag{30}$$

where $_{11}$ s_2 s_{23} and $_{12}$ s_3.

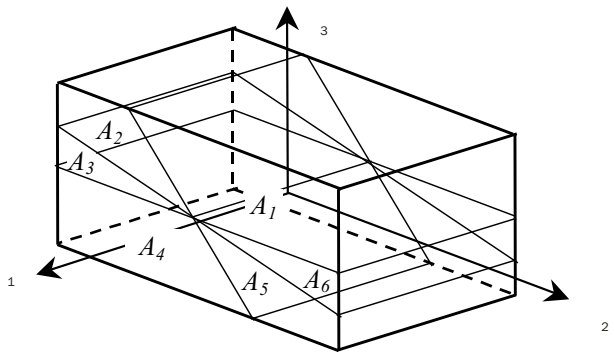

Figure 5. Aspects of the 3R manipulator

2. The 4R manipulator

The manipulator is shown in figure 6 and its geometric parameters are presented in table 2. The joint variables are θ_1, θ_2, θ_3 and θ_4, and its limits values are $\theta_i^{(l)}$ 150 and $\theta_i^{(u)}$ 150 , for i = 1, 2, 3, 4. To specify the linear velocity of the end effector we use the point O_5 of the frame $_5$ which is attached at the tip of the fourth link.

By considering the velocity vector of O_5 referred to the frame $_0$ as $\dot{\boldsymbol{x}}$ $[\dot{x},\dot{y},\dot{z}]^T$, the jacobian matrix of the 4R manipulator is:

$$\boldsymbol{J} \quad \begin{array}{ccc|ccc} (c_2 & c_{23} & c_{234})s_1 d & (s_2 & s_{23} & s_{234})c_1 d \\ (c_2 & c_{23} & c_{234})c_1 d & (s_2 & s_{23} & s_{234})s_1 d \\ & 0 & & (c_2 & c_{23} & c_{234})d \end{array}$$

$$\begin{array}{ccc|c} (s_{23} & s_{234})c_1 d & & s_{234}\, c_1 d \\ s_{23} & s_{234})s_1 d & & s_{234}\, s_1 d \\ (c_{23} & c_{234})d & & c_{234}\, d \end{array} \qquad (31)$$

The 3-order minors of the jacobian matrix which are non zero everywhere in the joint space are:

$$m_1 \quad d^3 (c_2 \quad c_{23} \quad c_{234})(s_3 \quad s_{34}) \qquad (32a)$$

$$m_2 \quad d^3 (c_2 \quad c_{23} \quad c_{234})(s_4 \quad s_{34}) \qquad (32b)$$

$$m_3 \quad d^3 s_4 (c_2 \quad c_{23} \quad c_{234}) \qquad (32c)$$

The conditions of configurations which render zero the above minors provide the following equations of the surfaces dividing the joint space in aspects:

$$c_2 \quad c_{23} \quad c_{234} \quad 0 \qquad (33)$$

$$s_3 \quad s_{34} \quad 0 \qquad (34)$$

$$s_4 \quad s_{34} \quad 0 \qquad (35)$$

$$s_4 \quad 0 \qquad (36)$$

Thus, twelve aspects can be identified (n_A=12) for the 4R manipulator. We chose the aspect A_1 for the achievement of the task; consequently, for inequality (17) we have k=1. Then we observe that the aspect A_1 is bounded by the surfaces of equations (33), (34) and (36); therefore for inequality (17) we have n_{hk} =3 and $\mathring{a}_{11}$=1, $\mathring{a}_{12}$=1, $\mathring{a}_{13}$=1. Thus, the implicit constraints on configurations to hold configurations in the aspect A_1 are:

$$_{11} \quad 0 \qquad (37)$$

$$_{12} \quad 0 \qquad (38)$$

$$_{13} \quad 0 \qquad (39)$$

where

$$_{11} \quad c_2 \quad c_{23} \quad c_{234}\,, \quad _{12} \quad s_3 \quad s_{34}$$

and

$$_{13} \quad s_4$$

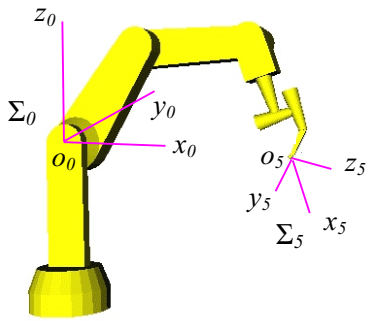

Figure 6. 4R Manipulator to be applied for three dimensional tasks

B. Single-objective problems

The tip of the 3R manipulator should complete a parabolic path; whereas in the case of the 4R manipulator, the tool has to describe a helicoidal path. The tasks of both robots are to be accomplished by applying cycloidal laws of motion to the end-

effector during 5 seconds. The Cartesian coordinates of the sample-points are referred to the frame $_t$; they are given in tables 3 and 5. In the two cases the path placements must be determined in such a way that the manipulability index is maximized on the point p_3 (when $t=2.5$ s). The independent variables of the planar problem are $r_{t}^{o}{}_{x}$, $r_{t}^{o}{}_{y}$, and (rotation about the axis $\boldsymbol{z_0}$); in the case of the 3D path, the additional variable $r_{t}^{o}{}_{z}$ must be included. The initial values for such variables, as well as the obtained optimal values are given in tables 4 and 6. In the same tables the initial and the optimal values of the objective functions are presented. The joint trajectories corresponding to the optimal placement are observed in figures 7 and 10. The behaviors of the normalized manipulability are compared in figures 8 and 11. Sequences of configurations of the robots during the accomplishment of the tasks are presented in figures 9 and 12 for the initial and the optimal placements.

B1. Parabolic path

Table 3. Coordinates of the sample path points in $_t$

Points	*x (m)*	*y (m)*
p_1	-0.2785	0.5028
p_2	-0.1392	0.3856
p_3	0.0000	0.3504
p_4	0.1392	0.3970
p_5	0.2547	0.4979

Table 4. Independent variables and objetive function

	r_{tx}^{0} *(m)*	r_{ty}^{0} *(m)*	*(degrees)*	Objetive function
Lower Limit	-1.0	-1.0	-90	-
Upper Limit	1.0	1.0	90	-
Initial Values	0	0	0	-0.1233
Optimal Values	0.0492	0.2405	-1.93	-0.1723

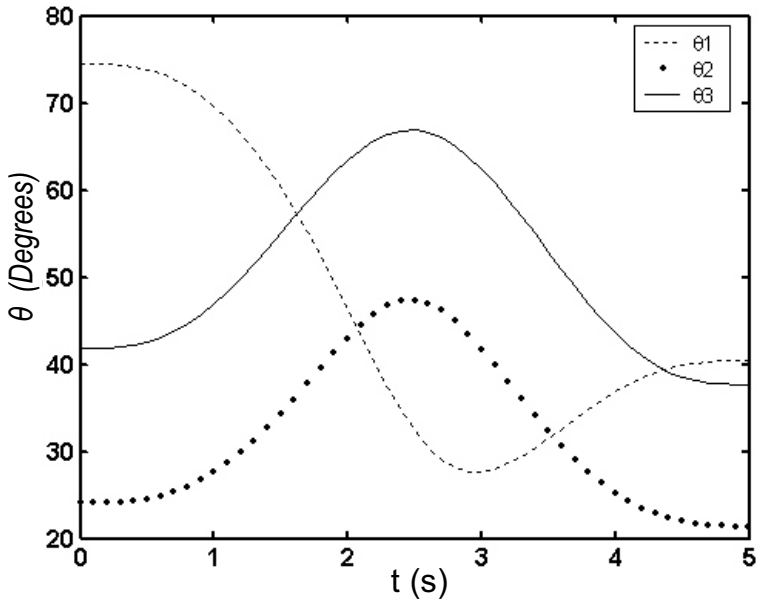

Figure 7. History of joint variables for the optimal placement. 3R manipulator. Single criterion

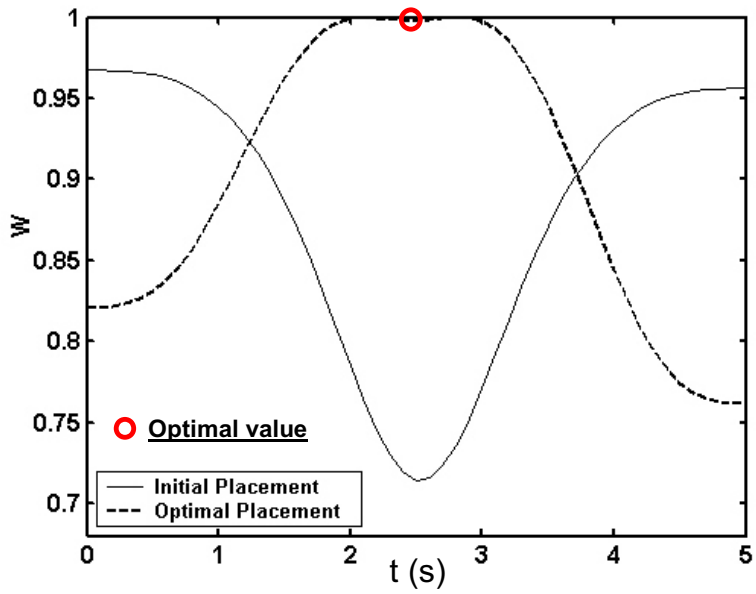

Figure 8. Normalized manipulability of the 3R manipulator for the planar task

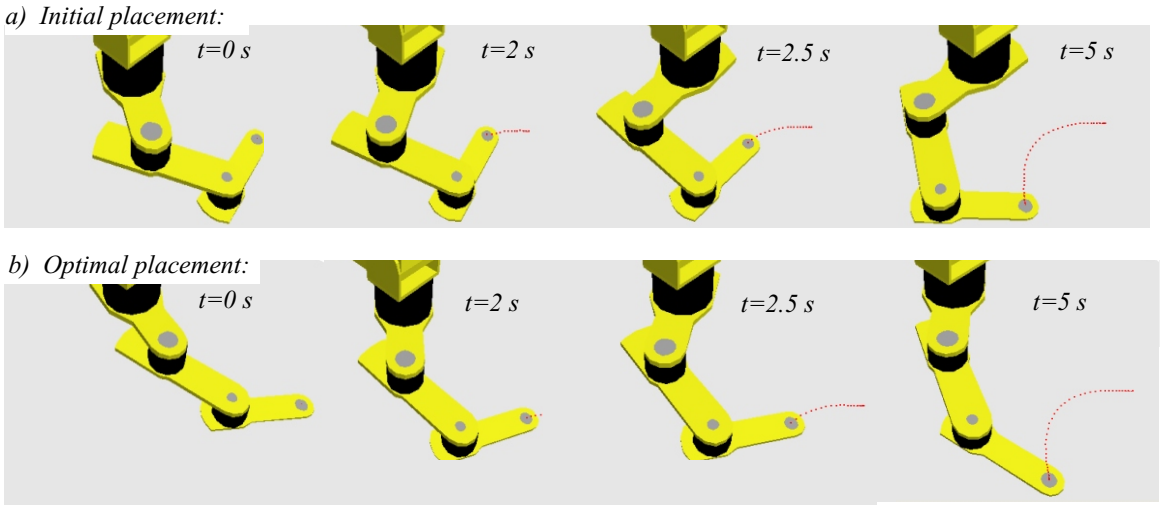

Figure 9. Simulation of the task

B2. Helicoidal path

Table 5. Coordinates of the sample path points in $_t$

Points	*x (m)*	*y (m)*	*z (m)*
p_1	0.1500	0.4000	0.0000
p_2	0.0464	0.5427	0.0307
p_3	-0.1214	0.4882	0.0613
p_4	-0.1214	0.3118	0.0920
p_5	0.1214	0.3118	0.138

Table 6. Independent variables and objetive function

	r^0_{tx} *(m)*	r^0_{ty} *(m)*	r^0_{tz} *(m)*	*(degrees)*	Objetive function
Lower Limit	-1.0	-1.0	-1.0	-180	-
Upper Limit	1.0	1.0	1.0	180	-
Initial Values	0	0	0	0	-0.1559
Optimal Values	0.0681	0.1142	-0.0572	9.91	-0.1639

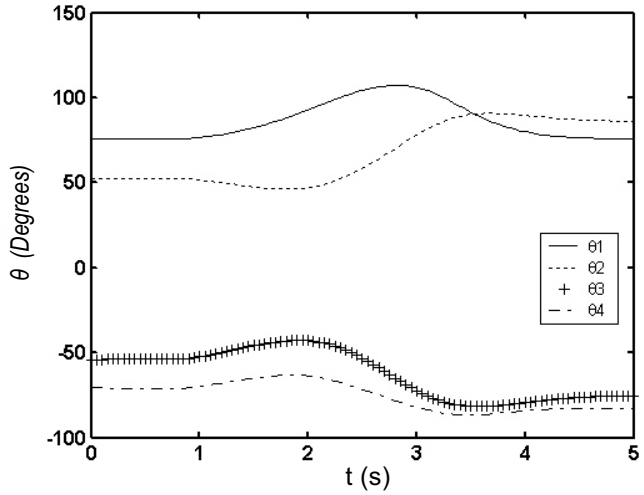

Figure 10. History of joint variables for the optimal placement. 4R manipulator. Single criterion.

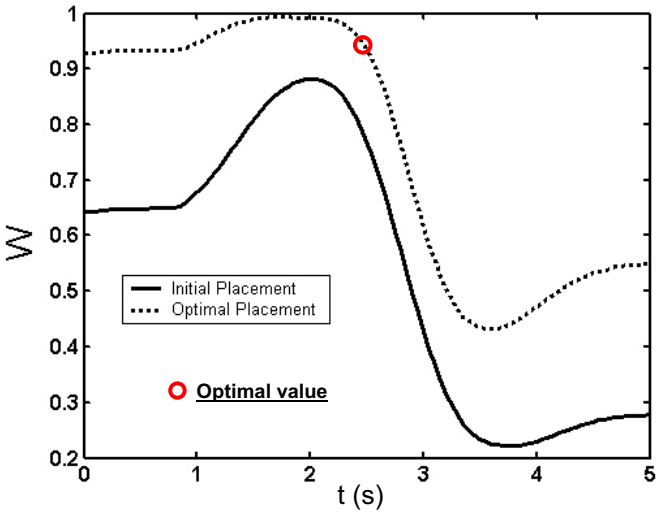

Figure 11. Normalized manipulability of the 4R manipulator during the task

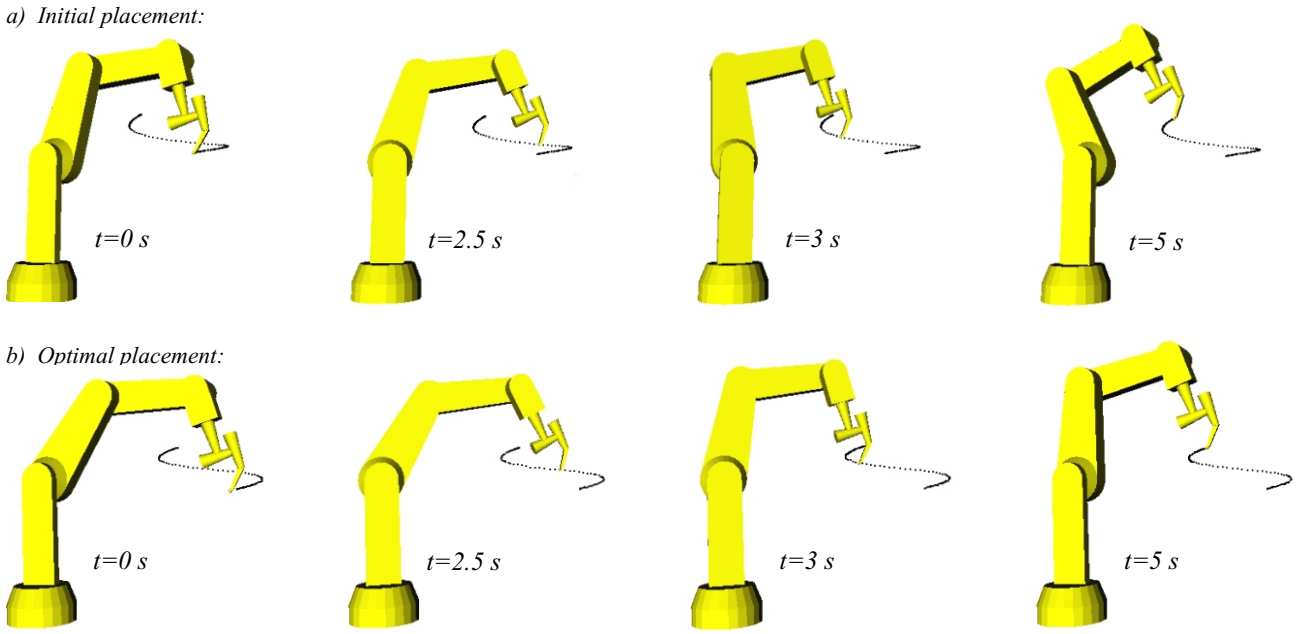

Figure 12. Simulation of the task

C. Multi-objective problems

In the case of the planar task the extremity of the 3R manipulator should complete a parabolic path; for the 3D task the 4R manipulator's tool has to describe a helicoidal path. In both cases the motion of the end-effector follows a cycloidal law. The periods of the tasks are 6 and 5 seconds for the 3R and 4R manipulators, respectively. The Cartesian coordinates of the sample-points are referred to frame $_t$, and different indexes of performance are associated to some points, as indicated in tables 7 and 10. In the problems we want to determine the path placement which allows to optimize the value of the specified indexes by the related manipulator's configuration when the task is carried out.

The normalization factors used in equation (18) are previously computed by solving the single-objective problem for points having a related index of performance. The obtained values of such factors are listed in tables 8 and 11. The independent variables of the planar problem are $r_{t_x}^{o}$, $r_{t_y}^{o}$, and (rotation about the axis $\boldsymbol{z_0}$); in the case of the 3D path, the additional variable $r_{t_y}^{o}$ must be included. The initial values for such variables, as well as the obtained optimal values are given in tables 9 and 12. In the same tables the initial and optimal values of the objective functions are presented. The joint trajectories corresponding to the optimal placement are observed in figures 13 and 18. The progress attained of the normalized indices associated to the sample points can be appreciated in figures 14 and 19. The behaviors of the manipulability and the condition number (both normalized) during the task are shown in figures 15, 16, 20 and 21. Sequences of configurations of the robots describing the desired paths are presented in figures 17 and 22 for the initial and the optimal placements.

1. Parabolic path

Table 7. Coordinates of the sample path points in $_t$

Point	*x (m)*	*y (m)*	Index of performance
p_1	-0.2785	0.5029	Manipulability
p_2	-0.2390	0.4614	-
p_3	0.0872	0.3701	Condition number
p_4	0.1614	0.4143	-
p_5	0.2424	0.4845	Condition number
p_6	0.2548	0.4979	Manipulability

Table 8. Normalization factors and time associated to the sample points

Point	Normalization factor	Value	*t (s)*
p_1	h_1^*	0.1727	0.0
p_3	h_3^*	1.0841	3.5
p_5	h_5^*	1.0841	5.0
p_6	h_6^*	0.1727	6.0

Table 9. Independent variables and objective function

	$r_{t\,x}^0$	$r_{t\,y}^0$	(degrees)	Objective function
Lower Limit	-1.0000	-1.0000	-90.0	-
Upper Limit	1.0000	1.0000	90.0	-
Initial Values	0.4000	0.1000	30.0	-0.5967
Optimal Values	-0.0206	-0.0378	-11.0	-0.8922

2. Helicoidal path

Table 10. Coordinates of the sample path points in $_t$

Point	*x (m)*	*y (m)*	*z (m)*	Index of performance
p_1	0.3722	0.1466	0.1126	Manipulability
p_2	0.3587	0.1770	0.1375	-
p_3	0.0935	0.3889	0.4005	Condition number
p_4	-0.2330	0.3251	0.6578	Condition number
p_5	-0.2554	0.3079	0.6790	-
p_6	-0.2876	0.2780	0.7120	Manipulability

Table 11. Normalization factors and time associated to the sample points

Point	Normalization factor	Value	t(s)
p_1	h_1^*	0.1526	0.0
p_3	h_3^*	1.4104	2.45
p_4	h_4^*	1.4104	3.75
p_6	h_6^*	0.1526	5.0

Table 12. Independent variables and objective

	r_{tx}^o (m)	r_{ty}^o (m)	r_{tz}^o (m)	(degrees)	Objective function
Lower Limit	-1.0000	-1.0000	-1.0000	-180.0	-
Upper Limit	1.0000	1.0000	1.0000	180.0	-
Initial Values	0.0000	0.0000	0.0000	0.0	-0.3537
Optimal Values	0.0865	0.1233	-0.2438	16.8	-0.6635

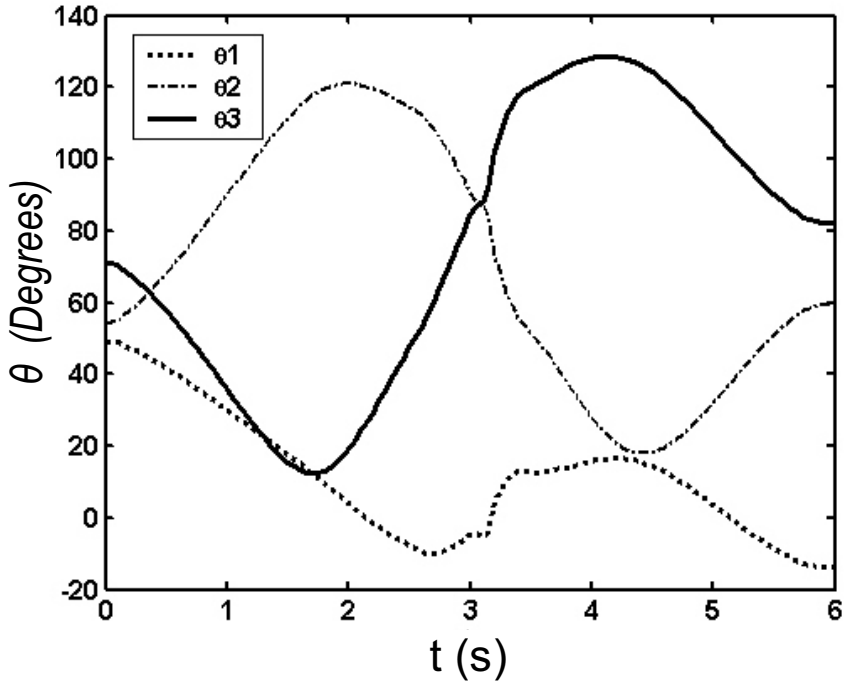

Figure 13. History of joint variables for the optimal placement. 3R manipulator. Multi-criteria

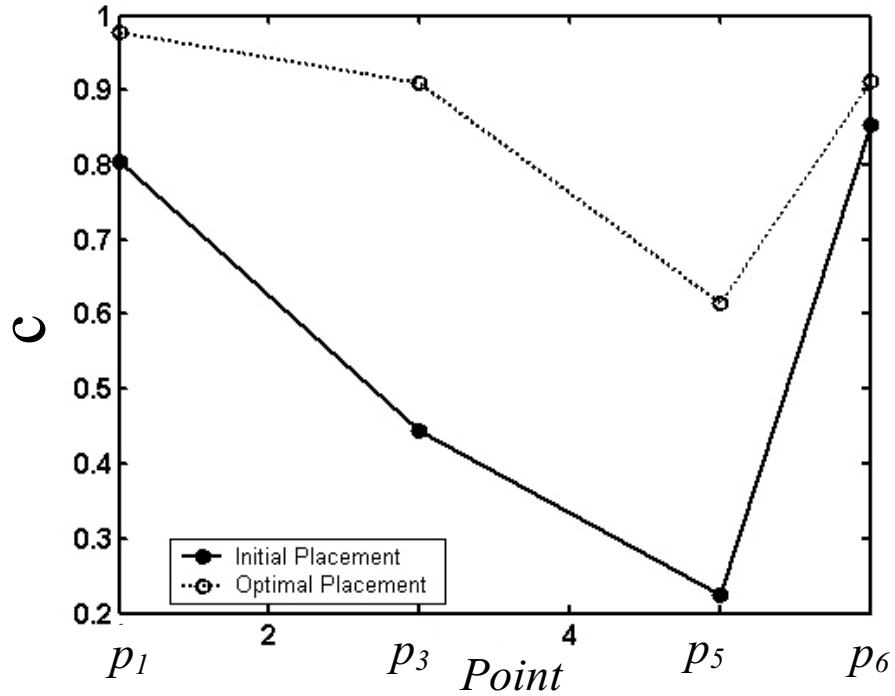

Figure 14. Values of the normalized indices

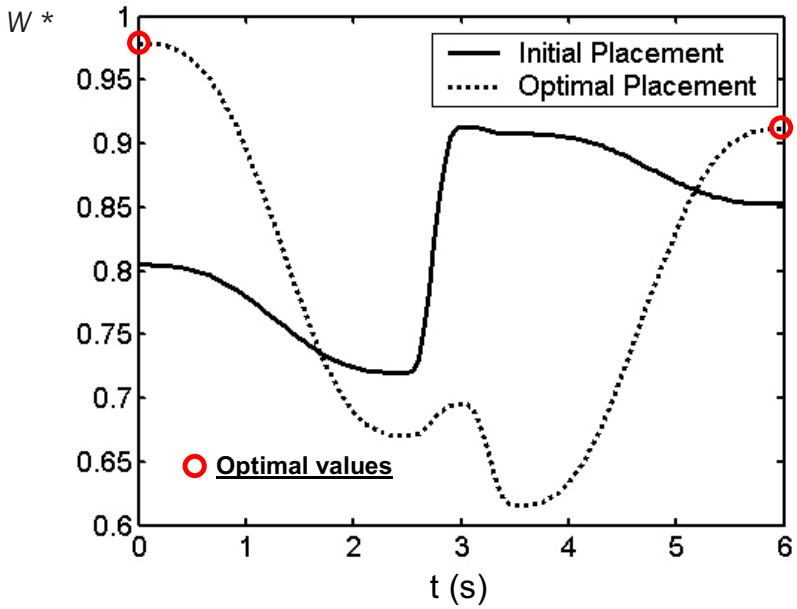

Figure 15. Behavior of the normalized manipulability during the task

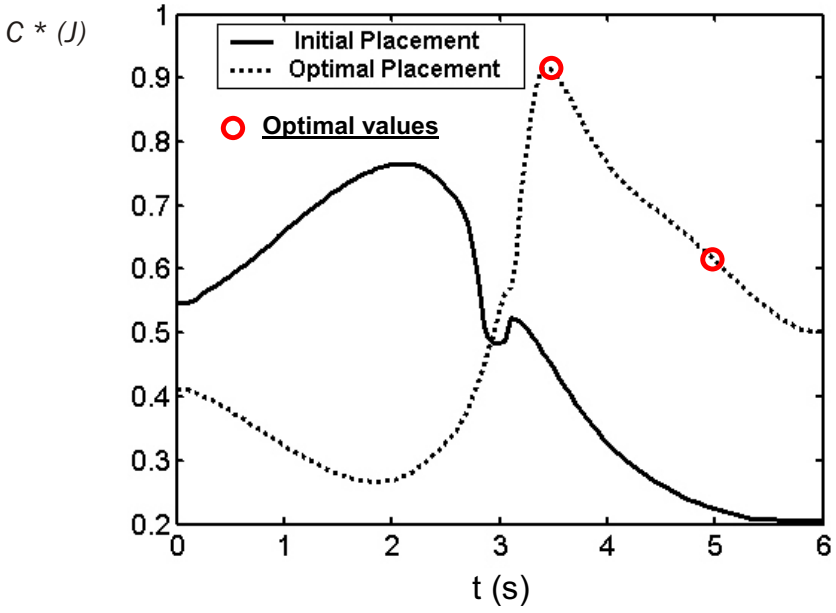

Figure 16. Behavior of the normalized condition number during the task

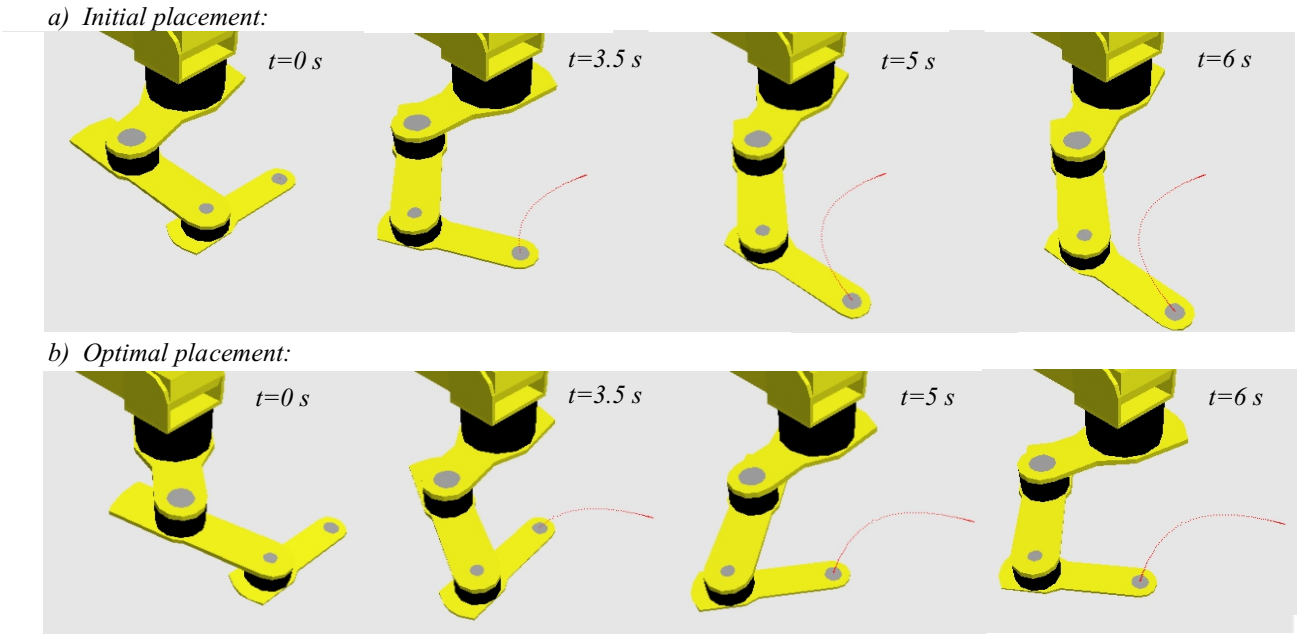

Figure 17. Simulation of the task

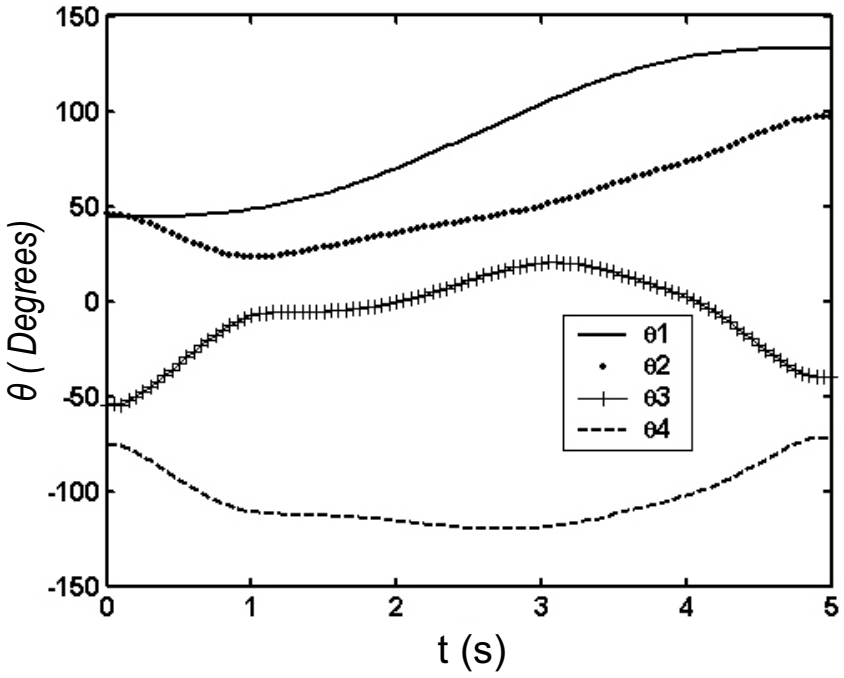

Figure 18. History of joint variables for the optimal placement. 4R manipulator. Multi-criteria

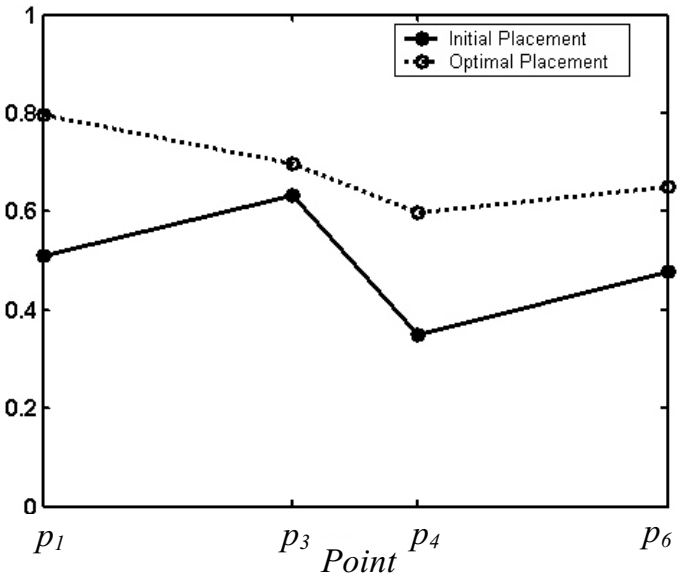

Figure 19. Values of the normalized indices

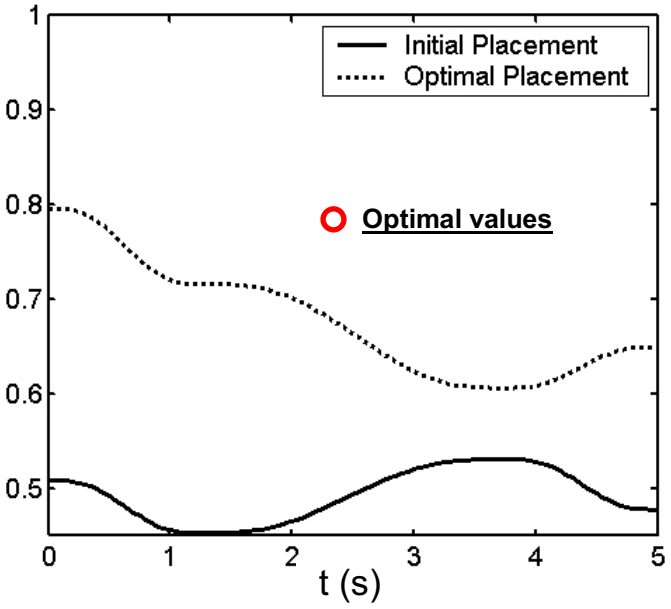

Figure 20. Behavior of the normalized manipulability during the task

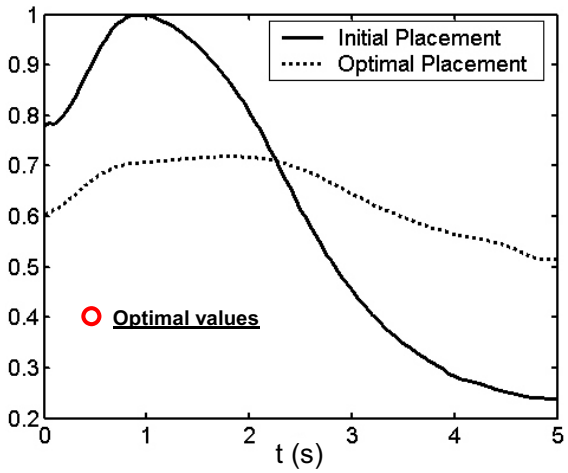

Figure 21. Behavior of the normalized condition number during the task

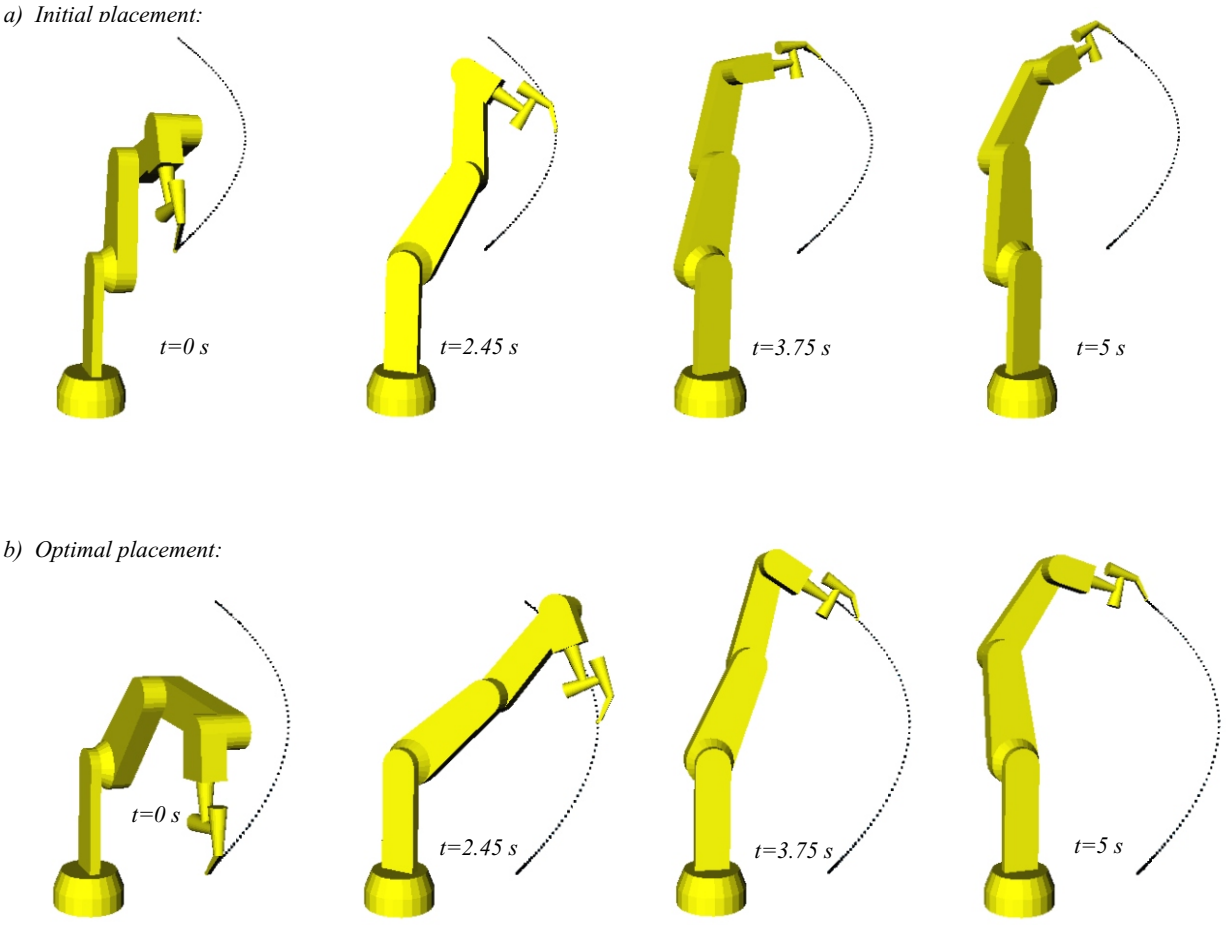

Figure 22. Simulation of the task

Conclusion

General formulations were presented in this paper to determine the best position and orientation of a path to be followed by a redundant robotic manipulator. Depending on the requirements of the user, the quality of a placement can be measured by using either a single criterion or multiple criteria of manipulator's performance. Consequently, the proposed formulations are addressed to solve both single and multi-objective optimization problems. In the single-objective problem, one index of performance associated to a specific path-point is defined as the function to be optimized. On the other hand, in the multi-objective problem such a function is equivalent to a characteristic index which represents the set of normalized indexes to be optimized. The proposed formulations take into account constraints regarding the accessibility to the manipulator's task. Indeed, we introduce constraints in order to: *a*) demarcate an available physical space to locate the task; *b*) avoid transgression of joint limits during the accomplishment of the task; *c*) generate continuous joint trajectories on the whole task.

The case studies examined here showed that significant improvements of the manipulator's performance can be obtained by applying our approach. In such cases all the constraints were satisfied and consequently the accessibility to the complete tasks was assured. However, in the hypothetical case in which a satisfactory solution could not be found by trying with a first mani- pulator's aspect, then further attempts could be accomplished by using other manipulator's aspects to satisfy the accessibility conditions and improve the manipulator's performance. On the other hand, the sample points considered in the problems were chosen by using a criterion of symmetry; nevertheless, both the number of points and the position of such points on the path could have an influence on the level of the improvement obtained for the indices of performance. Thus, supplementary studies should be carried out to characterize a suitable criterion to solve both questions. Furthermore, in future works additional constraints will be taken into account to avoid collisions of the manipulator when it works in cluttered environments.

Acknowledgments

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