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Force understanding through an in-lab learning proposal

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Force understanding through an in-lab learning proposal
Comprendiendo la fuerza: una propuesta de aprendizaje en el laboratorio

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Resumen

Un aprendizaje funcional de fuerza y tensión como cantidades vectoriales, requiere que los estudiantes tengan la habilidad de razonar acerca de las operaciones con vectores. En este artículo se presenta una colección de datos para describir algunas dificultades de los estudiantes relacionadas con operaciones de vectores y el concepto de tensión debido a una modificación en el laboratorio. Estos datos sugieren que a pesar de estas modificaciones, algunos estudiantes no reconocen la naturaleza vectorial de la fuerza y la tensión. Otros estudiantes no cumplen el conocimiento procedimental para reconocer la fuerza neta o la tensión. Además, describimos algunas dificultades procedimentales y de razonamiento observadas en el uso de vectores en el contexto de tensión a través de modificaciones en el laboratorio relacionadas con fuerza y tensión.

Palabras clave: Modificaciones instruccionales de laboratorio, dificultades conceptuales, fuerza y tensión.

Abstract

A functional understanding of forces and tension as vector quantities requires students' capability of reasoning vector operations. In this article, we present data describing students' conceptual difficulties with vector operations and tension after lab instructional modifications. These data suggest that despite lab instructional modifications, some students do not recognize the vector nature of force and tension concept. Other students do not have the requisite procedural knowledge to determine net force or tension. In addition, we describe some procedural and reasoning difficulties we have observed in students' use of vectors in the context of tension through lab modifications related to forces and addition of forces.

Keywords: Instructional lab modifications; conceptual difficulties with tension and forces as vectors.

INTRODUCCIÓN

Many introductory physics courses students have understanding problems with forces and tension during lecture and laboratory, especially when these two concepts are treated as vector quantities (Flores, Kanim, & Kautz, 2004). There is a tendency, even among capable students, to jump to force components immediately, and to resort to memorizing what these components are in specific cases rather than deriving them from the geometry of the problem (Duval, 2006). We conducted a previous investigation to justify students' learning problems related to force concept. We collected and analyzed data related to introductory physics courses students' understanding difficulties with the concept of tension force in the context of massless strings (Flores *et al.,* 2010). Students'

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learning problems showed several tension concepts misunderstanding. Some of these conceptual difficulties were: *1)* Students believe that the angle of the string is related to tension, *2)* Students believe that the proximity to the object is related to the tension, and *3)* Students use inappropriate compensation arguments to reason about situations where both, the angle and proximity change simultaneously. For example, students were asked the question shown in Fig. 1. It was asked as part of a laboratory pretest at NMSU (New Mexico State University) to compare the magnitude of the tension at two different points along the left massless string (Flores, 2006).

Figure 1. Question used to compare the magnitudes of the tension at two points on different sides of a massless and frictionless pulley
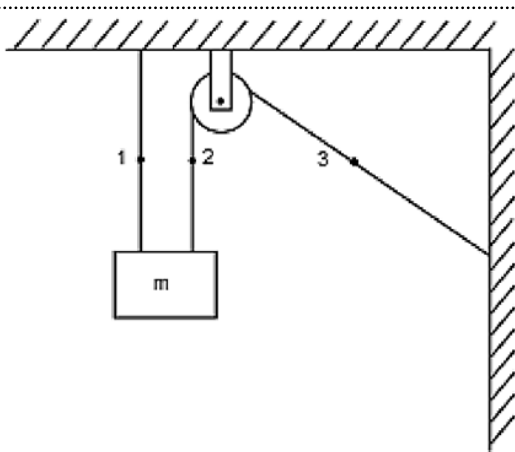

A block of mass M is hanging from two strings as shown in the figure. Is the magnitude of the tension at point 2 *greater* than, *less* than or *equal* to the magnitude of the tension at point 3?
Justify your answer.

The correct answer is that the magnitude of the tension at point 2 is equal to the magnitude of the tension at point 3, and does not depend on the angle on the relative positions of the points along the string.

Results showed that about one-half of the 94 students answered correctly. About one-fourth stated that the magnitude of the tension at point 2 was greater than at point 3 and one fourth said that it was less than the magnitude of the tension at point 3. About 20% of students used arguments about the difference between the angles that the string makes with the horizontal at points 2 and 3. An example of these responses is "Tension at point 2 is less because it has a greater angle at which the string is pulled."

Some understanding difficulties were associated to students' belief that the proximity to the object is related to the tension when students were asked the question shown in Figure 2. This question was given on examinations to 190 New Mexico State University (MNSU) students. They were asked to rank the magnitudes of the tension at five points in different sections of a massless string that is holding a block

Figure 2. Question to probe student understanding of tension on a massless string
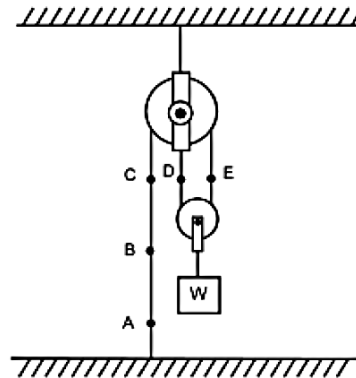
A block of weight W is suspended by two massless pulleys and a massless string as shown in the figure. Rank the magnitudes of the tensions at points A-E. Explain your reasoning.

The correct answer is since the string is massless, all of the points have the same tension. About 40% of students answered correctly. Almost one-half of the students who answered correctly explained that the tension is the same at the five points because they are on the same string.

Eleven of the thirty five students who answered incorrectly gave reasoning based on the idea that tension depends on the proximity of the points to the pulleys or to the hanging weight. For example one student answered that "Greater D, E, C, B, A least. Because the point closer to the weight will experience more tension." Another student stated that "*D=E>C>B>A. D* and *E* are supporting the weight and also have tension. C is next closest to the weight and also positioned at the top of the rope, where it is also being acted upon by tension B, then A is acted upon by tension and finally A." Other students based their reasoning on proximity to the pulley. For example, "Tension at point A is the greatest because it is farthest from the pulleys. Then followed by B, C and E. D has the smallest tension because it is closest to the pulleys."

Based on these results our question research is: *Is it possible that students improve force concept understanding through an in-lab learning proposal, based on a visualization of forces as vectors in tension context?* There-

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fore, the objectives of this investigation are: *1)* to address new students' learning effects of the exposition to lab activities by using low-range springs (probe sensors,) *2)* to develop a *change of representation* fluency related to the concept of tension, *3)* to explore students' problem difficulties with tension, and *4)* to create introductory mechanics courses curriculum.

PREVIOUS RESEARCH

Student understanding of tension and changes of representation

Some researchers have found that students use proximity reasoning as part of common misunderstanding when students need to change from one object's representation to another representation of the same object ((Knigth, 2013; Janvier, 1987, and Hitt, 1998). As part of an investigation into students' understanding of gravity, (Gunstone & White, 1981) 463 students were asked to compare the weight of a bucket with the weight of a block when the bucket and the block are hanging from a string stretched around a pulley as shown in Fig. 3.

Figure 3. The bucket and the block are suspended from a bicycle wheel
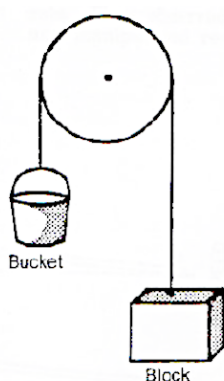
About one-half of the students concluded correctly that the weights are equal. About one-fourth stated that the block is heavier, and the most common reason for this response is that "the block is nearer to the floor."

Student understanding of vectors

Knight (1995) probed student's ability to: *1)* recognize and use vector components, *2)* assess the magnitude and direction of a vector, *3)* add two vectors graphically, and *4)* add two vectors using components. *The Vector Knowledge Test* was given to 286 students enrolled in a first-quarter calculus-based introductory physics courses. The test was administered before any instruction. Only 30% of students could write a brief definition of a vector, and only 43% of students could add two vectors graphically. About 15% could express a given vector as a magnitude and an angle. Knight (1995) concluded that beginning physics students need explicit instruction and practice with the use of vectors: Most of the students do not enter the introductory courses with enough vector knowledge to understand the basic principles of Newtonian mechanics.

Nguyen and Meltzer (2003) constructed a quiz containing seven vector problems, most of which were in graphical form. This quiz was administered to students in all introductory general physics courses taught in Iowa State University: Results were obtained from 2031 students, 721 students from algebra-based courses and 1310 students from calculus-based courses. About 60% of students from calculus-based and only 30% of students from algebra-based courses could correctly answer a two-dimensional vector addition problem. The most common error was the use of a "tip-to-tip" algorithm for finding a vector sum. A more detailed description of the tip-to-tip algorithm was found by Flores (2006).

Meltzer (2005) analyzed student's problem-solving performance on similar problems posed in diverse representations. He found that the proportion of correct responses given a verbal representation was consistently higher than with a diagrammatic representation. He found special difficulties when using vector representations in the context of Newton's third law. Many students showed a lack of understanding of the use of vector arrows to distinguish between forces acting *on* an object and forces exerted *by* the object.

Student understanding of tension and forces as vectors

To identify common student conceptual errors to recognize the existence of passive forces such as the tension

in a string, Sjoberg and Lie (1981) of the University of Oslo administered a written questionnaire to over 1000 secondary school students, future teachers, university students and physics graduate students.

Figure 4 shows two pendulums, one stationary and one swinging through its equilibrium position. Sjoberg and Lie (1981) asked students to indicate the forces acting on both pendulums. Results indicated that about 50% of the secondary-school students with one year of physics omitted the tension in the string. About 40% of the future teachers and about 10% of the graduate students omitted this force as well. A great number of students included a force in the direction of the motion of the swinging pendulum.

Figure 4. Experiment set used by Sjoberg and Lie to probe student difficulties with forces
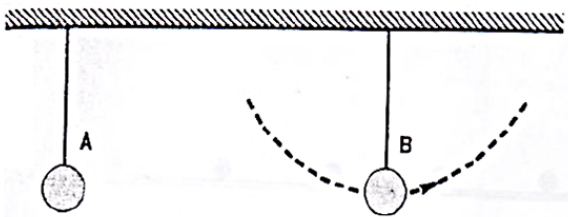
Methodology

Context of research (population)

The data presented here were collected at New Mexico State University (NMSU) from 2003 to 2004, and the University of Juarez in Mexico (UACJ) in 2005. The courses used as information sources for this investigation were:
  a) NMSU: Physics 215 (Introductory calculus-based mechanics).
  b) NMSU: Physics 211 (Introductory algebra-based mechanics).
  c) NMSU: Physics 215 laboratory.
  d) NMSU: Physics 211 laboratory.

Physics 215 is primarily intended for engineering majors. Instruction in introductory calculus-based physics courses at New Mexico State University consists of three 50-minute lectures. The sequence of topics in lecture follows the sequence in most textbooks. There is no recitation section.

Physics 211, the algebra-based physics course, covers more topics than the calculus-based course, but at a less rigorous mathematical level. The majors of the students enrolled in Physics 211 are approximately: 30% Engineering Technology, 30% Biology, 10% Agriculture, 5% Education, and 20% Other/Undeclared.

There is an associated 1-credit laboratory, Physics 211L and Physics 215L, required for some majors. About one-half of the students enrolled in the lecture portion of the course also take the laboratory. The 3-hour laboratory is graded separately from the lecture. All of the laboratory sessions are taught by graduate students. In laboratory, students work in small groups on materials intended to strengthen connections between observed phenomena and mathematical formalism, to promote scientific reasoning skills, and to foster conceptual understanding. Instead of a laboratory report, students are assigned laboratory homework intended to reinforce and extend concepts underlying the laboratory. Students are encouraged to *predict, compare* or *rank* variables in physical situations (Hieggelke, D.P., & T.L., 2004). Most of the laboratory sessions for both the calculus-based and the algebra-based course were based on *Tutorials in Introductory Physics* (McDermott, P., & Washington, 2002). We have modified these tutorials for use in the laboratory, and will describe these modifications in following sections.

The questions we have asked at the UACJ were in a 3-hour calculus-based physics class. A 90-minute laboratory session per week is required, but it is a separate course for grading purposes. The corresponding lecture instructors teach all laboratory sections. There is no recitation section. Most of the students are engineering majors. Questions on homework and in examinations are primarily quantitative.

Addressing student difficulties with vectors and tension

We described in the previous research various difficulties students have with the use of vector operations in the context of forces and acceleration (Flores, Kanim, & Kautz, 2004) and with the vector nature of tension in the context of massless strings in two dimensions (Flores, 2006). Our results suggest that many students fail to recognize Newton's second law as a vector equation. Furthermore, students who do recognize that they must

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treat kinematical and dynamical quantities as vectors often lack requisite procedural skills, or fail to recognize which features of a physical situation are relevant.

The results to the conceptual questions that we asked indicate that an understanding of Newton's second law as a vector equation was not a typical outcome of either the calculus-based or the algebra-based course. We wanted to investigate the extent to which we could improve student understanding of the vector nature of forces and of tension through laboratory-based instruction.

In this article we describe the modifications that we made to the mechanics laboratory sequence in order to improve students' ability to identify forces and draw free-body diagrams, and to reason about net forces. Included in these modifications are exercises intended to address specific difficulties that we have identified (Flores , 2006). Through a qualitative-quantitative investigation we describe results from questions that were asked in order to measure the effectiveness of these changes.

Description of laboratory modifications

Research-based instructional modifications have been published that are intended for use in recitation sections, or as modifications to the lecture portion of the course. For example, *Tutorials in Introductory Physics* (McDermott, P., & Washington, 2002) are a set of activities intended to be used in a 50-minute recitation section. Students work in groups on exercises intended to guide them toward the development of conceptual models underlying introductory physics topics. These exercises often address specific difficulties that have been identified by physics education researchers. At most universities that have adopted these materials, the *Tutorials* have supplanted a traditional recitation section where an instructor reviews traditional problem solutions with students. For lectures, both the *Active Learning Problem Sheets (ALPS)* kits developed by Alan Van Heuvelen (1995) and the *Peer Instruction* materials developed by Mazur (1997) promote conceptual development through group work as part of physics lectures.

At New Mexico State University (as at many other universities) there is no recitation section, and the content and approach taken in the lecture portion of the course is determined by the individual instructor. In order to address some of the difficulties related to tension forces

as vectors, we decided to introduce instructional modifications as part of the physics laboratories.

Prior to this investigation, an initial attempt to promote conceptual understanding through laboratory work had been made through the introduction of some of the materials from the *Tutorials*. The tutorials that are relevant to introduction of two-dimensional treatment of force and vectors that were used in modified form at New Mexico State University were *Motion in Two Dimensions, Forces,* and *Tension*. These materials were augmented with relevant laboratory exercises on the same topics.

Our original attempts at modification included all of this material in a single laboratory. However, as this investigation proceeded and we recognized additional student difficulties (Flores, Kanim, & Kautz, 2004), we expanded this material into a two-laboratory sequence. These modified laboratories are currently the fourth and fifth offered in the semester, and are titled *Forces* and *Addition of Forces*.

Based on our interviews and our analysis of questions asked to probe student understanding (Flores, Kanim, & Kautz, 2004), we have designed a laboratory sequence that includes:

1) Identifying the forces acting on an object and drawing free-body diagrams.
2) Exercises focusing on documented student difficulties with weight, tension, normal and friction force.
3) Pencil and paper practice with addition of vectors.
4) Qualitative and quantitative exercises that promote understanding of vector addition of forces.

Description of Force laboratory modifications

Each laboratory has an associated pretest, given at the beginning of the laboratory. Students are given 10 minutes to complete the pretest and are given a small amount of credit for completion. The pretest is not graded.

The pretests for the *Forces laboratory* and the *Addition of Forces* laboratories served to probe the ability of students to: *1)* recognize the forces acting on an object, *2)* compare the magnitude of the tension at various points along a massless string, *3)* add and subtract forces, *4)* rank the magnitudes of the forces acting on an object with no acceleration.

The first laboratory, *Forces* (Kanim, 2002) is intended to give students practice at identifying the forces acting

on an object and drawing free-body diagrams (this material is from the *Forces* tutorial), and to address key conceptual issues identified by our research and the research of others with regard to weight, tension forces, normal forces, and friction forces. Due to time limitations, we needed to restrict our focus to *specific issues* with regard to these forces.

The goals of our modifications to the laboratory were to promote through laboratory exercises: *1)* facility at translating from mass to weight, and of appropriate use of these quantities; *2)* recognition that the normal force acts in a direction perpendicular to a surface, and does not always have the same magnitude as the weight; *3)* recognition that the tension is constant along a massless or light string, and does not change when the direction of a string changes around a pulley; and *4)* identification of the friction force as dependent on the normal force and on the type of surface. Here we will describe only those parts of the laboratory related to tension.

Visualization helps students to develop an important conceptual understanding (Jewett, 2016) and (Flores, *et al.,* 2011). In attempting to address student difficulties with tension we used light springs inserted into strings in order to give a qualitative measure of the tension in the string in various places. Figure 5 shows the set of springs used in *Forces* laboratory. Students are told that the amount the springs stretch provides a measure of the tension in the string as a "visual tension sensor". Students are first asked to predict which spring will stretch the most when a mass is suspended, and to explain the basis for their prediction. Students test their prediction by adding slotted masses to the hanging hook as shown in Fig. 6.

As described through previous research (Flores, *et al.,* 2010), some students gave responses that are consistent with a belief that the tension in a string depends on its direction or orientation. In the *Forces* laboratory, we have students predict the relative stretches of the springs when the string is placed on a pulley as shown in Fig. 7. They are then asked to try this experiment and to resolve any inconsistencies.

Figure 5. Equipment set used for tension portion of the *Forces* laboratory
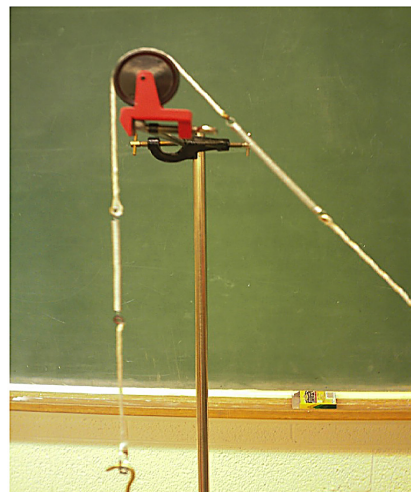
The homework associated with the *Forces* laboratory is intended to give students practice at applying the ideas developed in the laboratory. Students are asked about the ranking of the magnitude of the tension at several points along a massless string wrapped around two pulleys. An explanation of reasoning is required. Students are also asked to draw the free-body diagrams of situations related to objects resting on ramps, and the free-body diagram of a ladder that is resting against a wall. In addition, students are asked to calculate the magnitude of the gravitational force acting on specific masses, and the mass for specific weights.
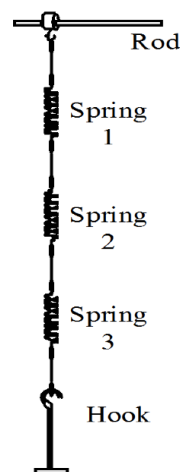
Figure 6. Linear spring set

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Figure 7. Linear set placed on a pulley
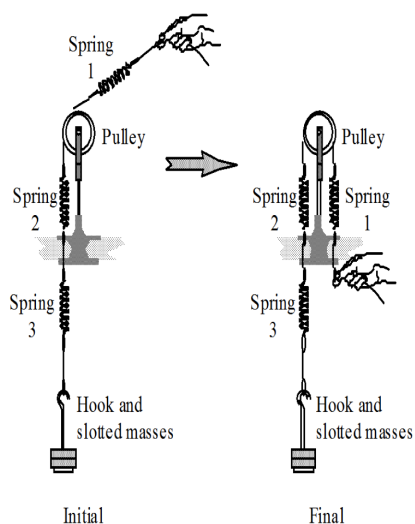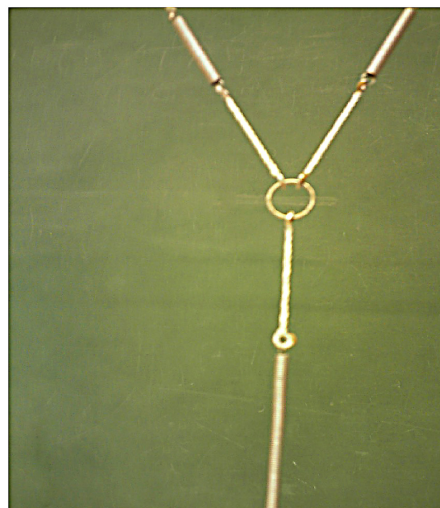

Figure 8. Equipment set used in *Addition of Forces* laboratory

Description of Addition of Forces laboratory modifications

The first part of the second laboratory of the two-laboratory sequence, *Addition of Forces,* is intended to give students practice with: *1)* addition of vectors, *2)* addition of forces included a free-body diagram, and *3)* qualitative reasoning about force magnitudes and directions for static cases. The issues we attempted to address are student tendencies to: *1) Close the loop* when adding vectors; *2)* Add vectors as scalars (Flores, Kanim, & Kautz, 2004); *3)* Inappropriately reason about tension based on angles; and *4)* Associate tension with string length (Flores, 2006).

The second portion of the laboratory is intended to give students practice with qualitative and quantitative reasoning about vector addition in static situations. In attempting to address student difficulties with tension, we used springs inserted into strings to give qualitative measure of the tension in the string at various places as in the *Forces* laboratory. Here however, three strings with springs are arranged in a "Y" as shown in Fig. 8. Students are first asked to draw a free-body diagram of a ring connected to three strings arranged in a vertical plane as shown in Fig. 9. Students are then asked to find the net force acting on the ring and to predict the relative lengths of the springs, and to then test their predictions.

Next, the angle between the upper strings is increased as shown in Fig. 10, and students repeat the procedure. Here they are asked to compare the tension in the strings to each other as well as to the tension in the previous case. Some students might conclude that the tensions in the upper strings do not change because the tensions are equal to one half of the hanging weight. Students are also asked to justify the answer. Some of them might draw a vector sum to compare the magnitudes of the tensions with respect to the angles of the strings.

Finally, this exercise is repeated for a situation where the upper strings make different angles with the vertical, as shown in Fig. 11.

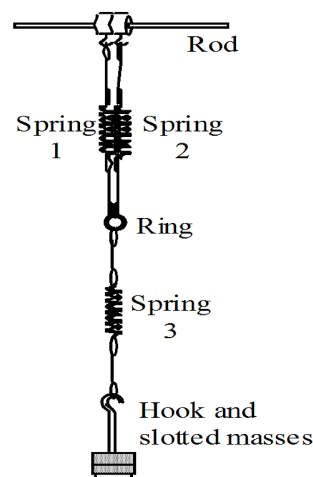**Figure 9. Linear spring set used in *Forces* laboratory**

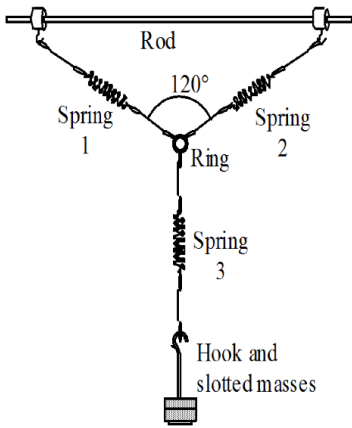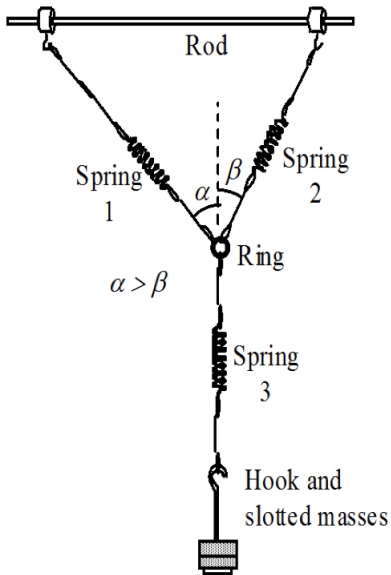**Figure 10. "Y" spring set.** *Addition of Forces* **laboratory**

**Figure 11. "Y" spring set with different angles**

In order to give students practice with the quantitative addition of forces, students then use a force table to observe the forces acting on a ring as shown in Fig. 12. They are first asked to draw a free-body diagram for the ring and to add graphically the forces from the free-body diagram. Students are then asked to predict the magnitude of the resultant vector. They then add force vectors representing the measured forces to find the magnitude and direction of the resultant force they found (which

should be zero). Finally, students are asked to find an unknown hanging mass by using a scaled vector sum. As with the *Forces* laboratory, homework is assigned to give students practice at applying the ideas developed in the laboratory.

We hoped that the modifications made to the *Forces* and *Addition of Forces* laboratories would improve student understanding of force and vector concepts. On completion of these laboratories, students from both algebra-based and calculus-based physics courses were given questions to measure the effectiveness of these modifications. These questions were included in laboratory homework and in a laboratory final examination. Homework and final examination questions were designed on a qualitative rather than a quantitative approach. In this posttest section, students have to use abilities such as prediction, ranking, and comparison of physical quantities.

Figure 12. Force table used in the
Addition of Forces **laboratory**
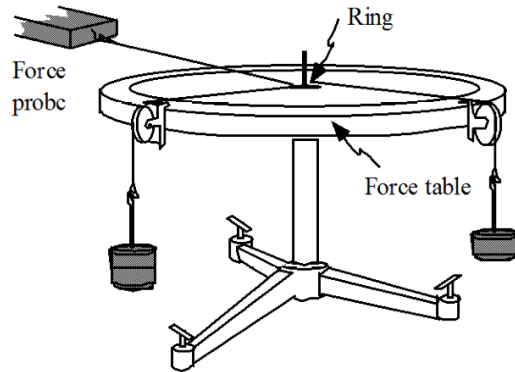
ASSESSMENT OF EFFECTIVENESS ON QUESTIONS ABOUT TENSION

Table 1 shows information related to the number of students and the type of exam to collect data. In addition, this table includes the course sections and the name of questions. Some students are maybe counted in different course sections.

The three question physical situation

As part of a laboratory final examination, the three multiple-choice questions shown in Fig. 13 were asked to 73 students from the algebra-based physics courses and

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65 students from the calculus-based physics courses. The correct answers are: *1)* For question 1 the tension at point A is *greater than* 1 N, *2)* for question 2 the tension at point E is *equal to* the tension at point A (because the pulley only changes the orientation of the string), and *3)* for question 3 the tension at point C is *equal to* the tension at point D.

About 95% of the students from both the algebra-based courses and the calculus-based courses correctly answered question 3. For comparison, after traditional instruction in lecture, about one-half of 122 students from the algebra-based courses and two-thirds of 112 students from the calculus-based courses correctly answered the question shown in Fig. 14 where students had to compare the magnitudes of the tensions at points 1 and 2 on a string located on the same side of a pulley.

Figure 13. Questions asked on the laboratory final examination
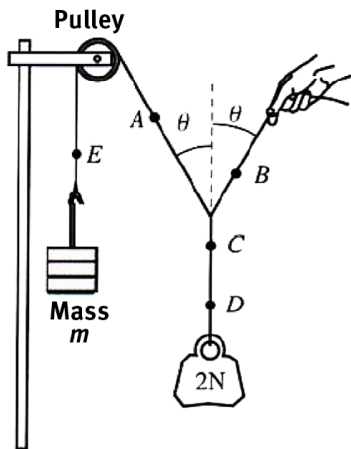
A 200 gram mass (weight 2 Newtons is suspended at rest as shown. All three threads are massless, and the pulley is frictionless and massless. The angle that the two upper ropes make with the vertical is the same (6) both cases.

1.- The tension at point A is:
  a) *less than* 1 Newton.
  b) *equal* to 1 Newton.
  c) *greater* than 1 Newton.

2.- The tension at point E is:
  a) *less* than the tension at point A.
  b) *equal* to the tension at point A.
  c) *greater* than tension at point A.

3.- The tension at point C is:
  a) *less* than the tension at point D.
  b) *equal* to the tension at point D.
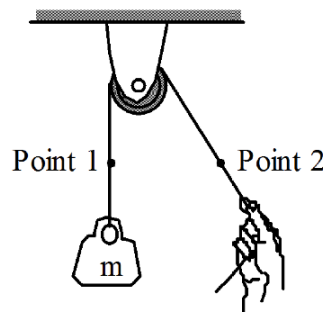 c) *greater* than tension at point D.

Figure 14 Question about the magnitude of tension at two points of a string
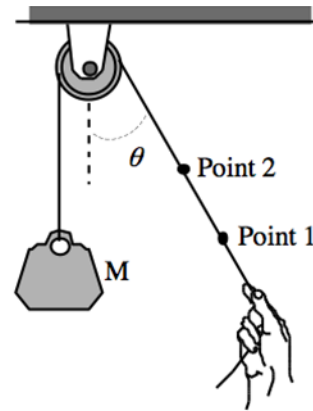
As shown at right, a student holds a massless string so that a piece of metal hanging from the other end of the string is at rest. The pulley is free to turn without friction. Is the magnitude of the tension at point 2 *greater* than, *less* than, or *equal* to the magnitude of the tension at point 1? Explain.

About 70% of students from the algebra-based courses and about 65% of students from the calculus-based courses correctly answered question 2. For comparison, after traditional instruction in lecture, about 70% from both the algebra and calculus-based courses correctly answered the question in Fig. 15 when it was asked as part of the *Forces* pretest (Table 1). For this question, students were asked to compare the magnitudes of the tension at two points in a string located on the two sides of a pulley. It appears that this portion of the laboratory was not very effective at improving student performance.

**Figure 15. Question about tension on opposite sides of a pulley**

As shown at right, a student holds a massless string so that a mass hanging from the other end of the string is at rest. The pulley is free to turn without friction.
Is the magnitude of the tension at point 1 *greater than*, *less than*, or *equal* to the magnitude of the tension at point 27 Explain your reasoning .

Table 1. Questions to assess tension understanding improvement

Type of exam	Question	Course sections	Number of students
Laboratory final exam (posttest)	The three multiple-choice questions (Fig. 13)	Algebra-based Calculus-based	73 65
Lecture final exam (posttest) traditional-based	The pulley question (Fig. 14)	Algebra-based Calculus-based	122 112
Lab exam (pretest)	The both sides of a pulley question (Fig. 15)	Algebra-based Calculus-based	73 65
Lecture Final exam (posttest) conceptual-based	The gymnast question (Fig. 16)	Algebra-based Calculus-based Calculus-based (UACJ)	73 65 105
Laboratory Forces Homework	The wrapped string question (Fig. 17)	Algebra-based Calculus-based	68 58
Lecture final exam (posttest) traditional-based	The five-point string problem (Fig. 18)	Algebra-based	30
Lecture Midterm exam	The five-point string problem (Fig. 18)	Algebra-based Calculus-based	100 60
Final examination	The five-point string problem (Fig. 18)	Calculus-based	105
Addition of forces Homework	The tilted-rod question (Fig. 19)	Algebra-based Calculus-based	93 46
Laboratory final exam (posttest)	The two different angle strings problem (Fig. 20)	Algebra-based Calculus-based	73 66
Addition of forces lab (pretest)	The two different angle strings problem (Fig. 20)	Algebra-based Calculus-based	

The reasoning required to answer this question is similar to the reasoning required to answer the gymnast question 1 shown in Fig. 16. About 40% of the student from the algebra-based courses and about the same percentage of students from the calculus-based courses correctly answered question 1. For the gymnast question, only 4% of the students in a final examination at the University of Juarez answered correctly. On a midterm examination at New Mexico State after modifications to the lecture portion of the course about 45% answered correctly. While a 40% correct response is not as good as we had hoped, results are about as good as if they were achieved through lecture modifications.

Figure 16. The static gymnast question
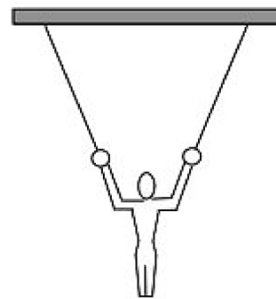
A 50-kg gymnast is suspended by two ropes as shown in the figure. Is the magnitude of the tension in the left string *greater than*, *less than* or *equal* to 250 N?
Consider *g=10 m/sec²*. Explain your reasoning

The wrapped string question

As part of the *Forces* laboratory homework, students were asked the question shown in Fig. 17, requiring them to rank the magnitudes of the tension at six labeled points along a massless string wrapped around two pulleys. The correct answer is that the magnitudes of the tensions at all six points are equal: We expected students give

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reasoning based on the fact that for situations with light strings and frictionless pulleys, a pulley only changes the direction of the sections of the string. Some students might consider the effect of the angle that the sections of the string make with a reference line. However, other students might recognize that the height of the pints does not affect the tension in the string.

Figure 17. Ranking questions about tensions at several points along a string
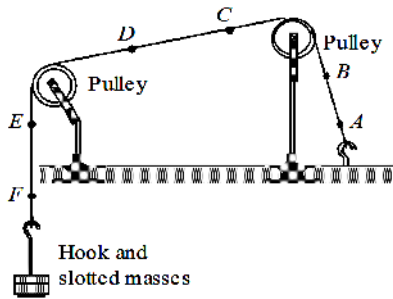
The massless string shown at right is wrapped around two pulleys. It is tethered to the desk on one side and a mass hangs from the other side. Rank the tensions at the six labeled points along the string. Explain how you determined your ranking.

About 75% of 68 students from the algebra-based course answered correctly. Eleven of the 52 students who answered correctly gave no reasoning. About 10% (7 students) reasoned based on the proximity of the points to the hanging mass. Four students gave a ranking of *A>B>C>D>E>F,* one of them explaining that "Point A supports the most weight," and 3 concluded that point F has the greatest tension. Two examples of explanations given are shown below.

F>E>D>C>B>A. Tension is greater at the point closer to the weight.

From greatest to least. F,E,D,A,B,C. F and E have more tension because they are holding the weight. B and C are second because they have the pulley to help A and B. A and B have two pulleys holding the weight.

About 70% of 58 students from the calculus-based course answered correctly. About 10% gave no response

and about 45% gave no answer justification. Only 20% of the students concluded that the pulley only changes the direction, not the tension. Only 4 of the 58 students concluded that each pulley absorbs some of the tension and gave the ranking *F=E>D=C>B=A*. One of these students stated "The tension should be less after each pulley."

As a comparison, we used the question described in Fig. 18 where students were asked to rank the magnitudes of the tension at points A-E. After traditional instruction, about 40% of 30 students from an algebra-based course at NMSU answered correctly. No students reasoned based on the proximity of the points along the string to a specific agent. Similarly, when this question was asked on a midterm examination, about 40% of 100 students from an algebra-based course at NMSU correctly answered the multiple-choice question shown in Fig. 18.

Figure 18. Multiple-choice version
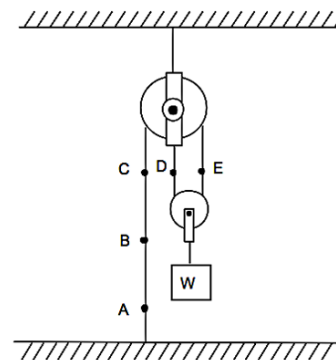
A block of weight W is suspended by two massless pulleys and a massless string as shown in the figure. The correct ranking for the magnitudes of the tensions at points A-E is:

a) A>B>C>D>E
b) A=B=C=D=E
c) A<B<C<D<E
d) A=B>C=D=E

On a midterm examination at NMSU, about 45% of 60 students from a calculus-based course answered this question correctly. Only five students reasoned incorrectly on the basis of the location of the points along the string. On a final examination at the University of Juarez, only about 35% of 105 students from calculus-based course answered this question correctly. This comparison suggests that the laboratory has contributed to students' understanding of tension along a massless string. However, the results from the similar final exa-

mination question were about the same as the results on similar pretest questions. As part of the development of this laboratory, further assessment of students' pre- and post-test performance on similar questions will be asked and analyzed.

The tilted rod question

An important goal of the *Addition of Forces* laboratory is to promote conceptual reasoning about the magnitudes and directions of forces in static situations. We recognize, however that this reasoning requires an understanding of individual forces, of the addition of these forces, and of the effect of changing angles and magnitudes of individual forces. In some sense, then, questions requiring reasoning about relative magnitudes of forces serve as "capstone" questions. Incorrect answers can result from a variety of sources in a chain of reasoning that many students find challenging. Here, we describe two questions that we have asked in order to assess the degree to which students are able to reason about relative forces in statics.

As part of the *Addition of Forces* homework, the question shown in Fig.19 was administered to 93 students

Figure 19. Question to assess student ability to add and rank tensions in strings
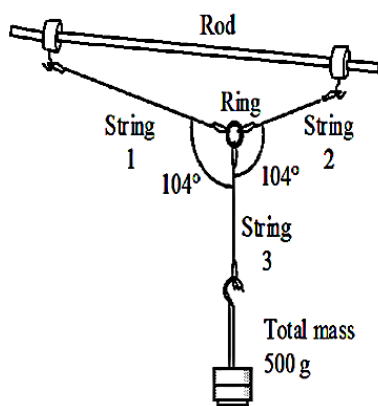
A total mass of 500 grams hangs from a string attached to a ring. Two other strings are stretched between the ring and the tilted rod as shown. String 1 is twice as long as strings 2 and 3 (which are the same length). Strings 1 and 2 make an angle of 104 with string 3. Use a free·body diagram for the ring and a graphical vector sum to rank the tension in the 3 ropes. Explain how you determined your answer.

from the algebra-based courses and to 46 students from the calculus-based courses. Students were asked to rank the magnitudes of the tensions in the strings after drawing a free-body diagram for the ring and to use the drawing to generate a graphical vector sum. The correct ranking is $T_1=T_2>W$ because the angles that strings 1 and 2 make with the vertical are equal each other and almost equal to 90°. We expect some students give the correct answer by drawing a vector sum with the same directions of the tension from the free-body diagram. However, some of them might draw the magnitude of the tension similarly to the lengths of the strings.

Students in the algebra-based and in the calculus-based sections performed similarly on this question, and we report their results together. Only about 30% of the students from both the algebra-based and the calculus-based courses answered correctly: All of these students also justified their responses correctly. About sixty percent of the students answered incorrectly and the rest of them gave no responses. Although most of the students who answered incorrectly drew correct free-body diagram, almost all of them gave a vector sum with forces in incorrect directions.

Some of the incorrect responses are: *1)* about 20% of all the students gave the reasoning $T_3>T_2>T_1$, *2)* about 5% gave the ranking $T_1>T_2=T_3$, *3)* about 7% concluded that the ranking is $T_1>T_3>T_2$, *4)* about 3% of all the students considered that the ranking is $T_1>T_2>T_3$, and *5)* the same percentage (3%) gave the ranking $T_3>T_1>T_2$.

As part of a laboratory final examination, 73 students from the algebra-based courses and 66 students from the calculus-based sections were asked the question shown in Fig.20. The correct answer is choice *d)*.

About one-half of students from both the algebra-based and the calculus-based courses answered correctly. About 15% of the students from the algebra-based courses gave the ranking 1>2>3; about 10% responded that all 3 points have the same tensions. No students from the calculus-based sections gave this response

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Figure 20. Question to assess student understanding of tension in a two-dimensional situation with three strings

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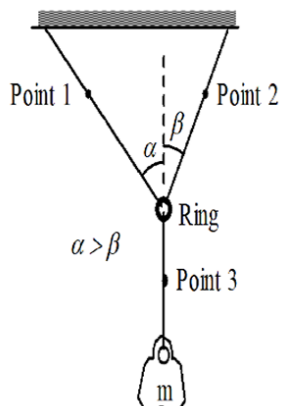
The 3 strings in the arrangement shown at right are massless, and the mass of the ring can be ignored. Which of the following is a correct ranking for the tensions at the labeled points?

    a) Point 1 is greatest, then point 2, then point 3.
    b) Point 1 equal to point 2, point 3 is smallest.
    c) Point 3 greatest, point 2 and point 1 smaller
       and equal to each other.
    d) Point 3 greatest, then point 2, then point 1.
    e) Point 3 greatest, then point 1, then point 2.
    f) All 3 points have the same tension.

In comparison, when this question was asked as part of *the Addition of Forces* pretest, about 35% of the students from the algebra-based sections and about 40% of the students from the calculus-based sections answered correctly. About 10% of the students from algebra-based sections and only about 5% from the calculus-based sections gave the ranking 1>2>3. About 20% of the students from the algebra-based and about 10% from the calculus-based sections said that the three tensions have the same magnitude. Only few of the students from both the algebra-based and the calculus-based sections used a graphical vector sum to compare the tension in the strings. Some of them drew the vector sum with incorrect angles. Other students said that the tension at point 3 is the greatest because this string is holding the weight.

ASSESSMENT OF EFFECTIVENESS OF INSTRUCTION IN THE CONTEXT OF ADDITION OF FORCES

Table 2 shows information related to the number of stu-

dents and the type of exam to collect data about addition of forces. In addition, this table contains the course sections and the name of questions. Some students may be counted in different course sections.

**Table 2. Questions to assess addition of forces understanding improvement**

Type of exam	Question	Course sections	Number of students
Laboratory Final exam (posttest)	The three-vector question 1 (Fig. 21)	Algebra-based Calculus-based	100 77
Addition of forces pretest	The three-vector question 2 (Fig. 21)	Algebra-based Calculus-based	100 77

As part of a laboratory final examination, students were asked to add the three forces shown in Fig. 21, and to then choose an answer from among the choices given. Choice a is the correct direction, but it is the wrong length, and choice c is the correct length but it is not in the right direction. Choice b is opposite to the resultant, and would result from "closing the loop." This misconception was found by Flores, Kanim, & Kautz (2004). Therefore, a correct answer is d. Different types of misconceptions were found by DiSessa (1993), McCloskey (1980), and Hammer (1996).

About 35% of the students from the algebra-based courses answered correctly, and about 30% of them "closed the loop" to add the three forces. About 60% of the students from the calculus-based courses answered correctly, and only about 20% of them "closed the loop" to add the forces.

Figure 21. Laboratory final examination question to address addition of forces

...

In the space below, use a ruler and a protractor to add the forces shown at right. Then choose the best answer for the vector sum from the choices given below.
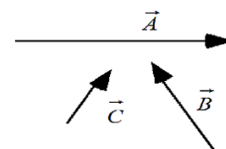
The question shown in Fig. 22 was asked as part of the *Addition of Forces* pretest. Students were asked to find the resultant for the addition of vectors $\vec{A}, \vec{B}$, and $\vec{C}$.

Figure 22 Laboratory pretest examination question to address addition of forces
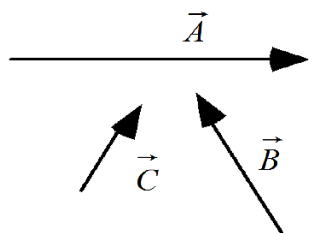
Find the resultant for the addition of vectors $\vec{A}, \vec{B}$, and $\vec{C}$. in the space provided below. Show your work

Similar responses were given by the students in both the algebra-based and calculus-based sections. About one-half of the students answered correctly. Only a few of the students (6 of 177 students) who answered incorrectly "closed the loop" to add the three forces (Flores, Kanim, & Kautz, 2004).

Results from the final examination question are actually *worse* than pretest results, especially for students in the algebra-based course. We suspect that some of this can be explained by the format of the final examination question: Students are generally hesitant to choose a 'none of the above' response. Nonetheless, the results are poor, and it may be that additional modifications are necessary to reinforce basic ideas about vector addition.

Conclusions

Through instructional modifications to laboratory, we hoped to promote an improved conceptual understanding of physical quantities such as forces, tensions, and their vector nature. Results after modified laboratories (*Forces* and *Addition of Forces)* seem to indicate improvement in students' responses for some of the questions asked in the laboratory homework and on the final examination. For example, the percentage of students who answered question 1 correctly in the situation shown in Fig. 13 (40%) is greater compared to the percentage (only

4%) who answered the gymnast question in the University of Juarez. This improvement is similar to what was obtained through modifications to lecture instruction.

Other responses in the context of tension indicate the same percentage of students who gave correct responses on both pretest and post-test. For example, the percentage of students who answered question 2 correctly when it was asked in homework (Fig. 13) is similar to the percentage of students who answered the pretest question shown in Fig. 15.

In general, we saw somewhat improved performance on questions that probed students' understanding of vector addition or that were focused on a specific conceptual difficulty. However, for questions such as that shown in Fig. 20 that require a general understanding of a multiple-step reasoning process related to statics, we did not see much improvement. While we will continue to modify the laboratory in hopes of improving student performance on similar questions, It is likely that success on questions like this require more experience and practice that can be provided in one or two laboratories. Therefore, a possible reason of some of the understanding problem found in this investigation is a lack of change in mathematical representations fluency (Knight, 1995), (Megowan-Romaniwicz, 2016) and (Ibarra, Flores, & Gonzalez, 2016).

Some limitations of this investigation are: *1)* the diversity of sections to collect data, *2)* absence of selection related to the academic students' level at the beginning of the study, *3)* absence of more conceptual-based specific interviews, and *4)* a lack of statistic indicators. Finally, in the future we will address these scopes by designing and a new statistical-based-lab investigation. The corresponding learning activities will help address students' understanding difficulties related to tension through the use of technology. We will compare both approaches, spring-based and PASCO force sensors. In addition, we will collect data to develop statistics based on indicators as gaining, abstraction levels and learning transfer (Flores, Gonzalez, & Victor, 2014) and (Ibarra, Flores, & Gonzalez, 2016).

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